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Production Cost Optimization in Sugar Manufacturing Using Linear Programming: A Case Study of Savannah Sugar Company Limited, Numan, Nigeria

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ABSTRACT

Sugar manufacturers in Nigeria work under fluctuating cane supply, limited capacity, costly overtime, high electricity cost and limited storage, yet monthly production is often planned by experience alone, which can lead to unnecessary overtime, unnecessary stock and higher cost. This study develops a linear programming (LP) model to minimize the production cost of Savannah Sugar Company Limited (Dangote Sugar Refinery), Numan, Nigeria, over six months from January to June 2025. The model has 18 decision variables and 66 constraints on demand balance, production capacity, raw material, labour, machine hours, electric power and storage, and electricity is charged to each ton produced. Data came from company records and interviews with 13 key informants, with labour, machine-hour, electric power and storage parameters assumed where records were unavailable, and the model was solved with the Simplex LP engine of Microsoft Excel Solver, followed by sensitivity analysis. The minimum total cost is ₦4,678,873,844.50. The optimal plan produces 16,174.6 tons in regular time and 1,595.4 tons on overtime, uses overtime only from April to June, and leaves no stock at the end of June. Regular capacity and raw material are the binding constraints: one more ton of regular capacity saves ₦25,000 to ₦53,142.50, and one more ton of raw material saves ₦2,185.75 to ₦7,185.75 in April to June. A break-even holding period of about 2.1 months explains the timing of overtime. Tests on the assumed parameters show that the plan is unchanged for adverse changes of at least 10% in machine time and storage but is sensitive to the regular-time labour and electric power limits, where a change above 1.9% brings overtime forward and raises cost by about 1% to 4%. The study concludes that LP with Excel Solver is a practical planning tool for a Nigerian sugar plant, provided that the assumed parameters are replaced by measured plant values.

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1. Introduction

A sugar mill sits between the farm and the market. Each month its output depends on how much cane arrives, how many labour and machine hours are available, how much electric power the plant can draw, and how much sugar it can store. Savannah Sugar Company Limited (SSCL), now part of the Dangote Sugar Refinery group, works within all of these limits at Numan, together with an unsteady cane supply, expensive overtime, high electricity costs and variable demand. When monthly output is set by experience alone, a plant

tends to run more overtime than it needs, hold stock it does not need, and leave capacity either idle or overstretched. All of these raise cost.

During our visits and interviews, staff described monthly production as being set mainly by experience and discussion between the production and finance units. We therefore asked whether a simple optimization model could guide the monthly decision.

Linear programming (LP) is the standard method for allocating limited resources among competing activities at least cost (Dantzig, 1963; Hillier & Lieberman, 2015). In Nigeria it has been applied to product-mix and production planning problems in beverage (Danfulani *et al.*, 2022), detergent (Amole *et al.*, 2016), paint (Adebiyi *et al.*, 2014), bakery (Ailobhio *et al.*, 2018), bottling (Njoku & Gambo, 2022) and dairy (Ahmed & Ogbada, 2026) firms. Studies of aggregate production planning elsewhere, including those of Brazilian sugar and ethanol mills (Paiva & Morabito, 2009; da Silva & Marins, 2014), show that the best plan depends on the balance between regular-time production, overtime and inventory.

In the studies reviewed here, however, we found no LP application to a Nigerian sugar plant that plans regular-time production, overtime and inventory together while charging electric power to each ton produced and limiting it by availability. This gap matters because management needs to know when overtime is worth its premium, how long stock should be held, and what an extra ton of cane or capacity is worth, and the shadow prices of an LP model answer these questions directly. The objective of this study is therefore to develop and solve an LP model of monthly production planning at SSCL, Numan, that finds the least-cost plan and uses sensitivity analysis to identify the constraints that matter most.

2. Literature Review

Aggregate production planning (APP) determines production and workforce levels for each period of a medium-term horizon so that fluctuating demand is met at least cost (Cheraghalikhani *et al.*, 2019). Early formulations used quadratic costs (Holt *et al.*, 1960), the transportation method (Bowman, 1956) and linear programming (Hanssmann & Hess, 1960). Cheraghalikhani *et al.* (2019) list regular-time and overtime production and inventory carrying among the basic cost elements of APP models, which are the elements modelled here. Later reviews have organized the field by model structure and solution method (Cheraghalikhani *et al.*, 2019), by the treatment of uncertainty (Jamalnia *et al.*, 2019), by sustainability and circularity across about 200 papers (Aydin & Tirkolaee, 2022), and by social and economic perspectives (Leong *et al.*, 2025).

In the sugar industry, Paiva and Morabito (2009) built a mixed-integer model for the aggregate production planning of a Brazilian sugar and ethanol milling company, and da Silva and Marins (2014) proposed a fuzzy goal programming model for a Brazilian sugar mill that covers sugar, ethanol, molasses and derivatives under uncertainty over a weekly planning horizon. These studies deal mainly with cane supply, milling decisions and the product mix.

Nigerian applications of LP are mostly product-mix or profit-maximization studies. Adebiyi *et al.* (2014) used LP with LINDO software to select the product mix of a paint producer, Ailobhio *et al.* (2018) maximized profit at a bakery in Lafia, and Danfulani *et al.* (2022) applied LP to production planning at a beverage company in Yola. Njoku and Gambo (2022) modelled production, inventory and distribution at a bottling company and reported a 5% profit improvement, and Ahmed and Ogbada (2026) used the simplex method for yoghurt production at a dairy farm. The closest study to ours is Amole *et al.* (2016), who formulated an LP model of production planning at a detergent firm with regular-time and overtime production costs and inventory costs, and reported that its monthly cost of production could be reduced.

Recent issues of this journal carry related optimization work. Ahmed *et al.* (2026) built a multi-objective LP model for vendor selection in the agro-allied sector of Ganye and Tougo, Adamawa State, and used sensitivity analysis to show that its gains in rejected units and late deliveries hold over a wide range of weights. Onyenike *et al.* (2026) developed a fuzzy inventory model in which shortages are not allowed and used sensitivity analysis to examine how changes in the parameters affect inventory cost, and Stephen *et al.* (2026) applied an artificial bee colony algorithm to the inventory routing problem, which minimizes delivery cost relative to the volume delivered. Two of these studies relax the single-objective or deterministic setting of the earlier Nigerian studies, with multiple objectives (Ahmed *et al.*, 2026) and fuzzy parameters (Onyenike *et al.*,

2026), but none of them addresses the monthly trade-off between regular-time production, overtime and inventory that is modelled here.

Taken together, the global reviews and the Nigerian studies differ in emphasis. The global literature has moved toward uncertainty (Jamalnia *et al.*, 2019), sustainability and circularity (Aydin & Tirkolaee, 2022) and social as well as economic criteria (Leong *et al.*, 2025), whereas the Nigerian studies reviewed here are mostly deterministic and single-objective and were solved with software such as LINDO, TORA or R (Adebiyi *et al.*, 2014; Ailobhio *et al.*, 2018; Danfulani *et al.*, 2022). The present study sits between the two. It keeps the transparent, spreadsheet-based approach that plant managers in Nigeria can use themselves, but it adopts the regular-time, overtime and inventory structure and the energy cost that the global aggregate planning literature treats as basic, and Section 5 shows how uncertainty can be added in later work.

Excel Solver provides a Simplex LP engine and a sensitivity report (Fylstra *et al.*, 1998), and standard texts explain how to read its output (Winston & Goldberg, 2004; Hillier & Lieberman, 2015; Taha, 2017). In summary, the studies reviewed here do not combine regular-time production, overtime and inventory with an electric power cost and limit for a Nigerian sugar plant, and few of them interpret shadow prices in economic terms. This study addresses that gap.

3. Methodology

This section explains how the production planning problem at SSCL was turned into a linear program and solved. Section 3.1 describes the study area and the data, Section 3.2 gives the general LP formulation, Section 3.3 sets out the six-month model with its notation, objective function and constraints, and Section 3.4 describes how the model was implemented and analysed in Excel Solver.

3.1 Study Area and Data

The SSCL is in Lamurde Local Government Area of Adamawa State, north-eastern Nigeria. The company grows sugarcane and makes sugar seasonally, crushing cane over a harvest window of roughly five to six months. We used a quantitative single-case design (Yin, 2018). Our data were the company's monthly records for January to June 2025 (production, costs, raw material, labour, machine hours, electricity, inventory and sales), the latest complete period available, and interviews with 13 key informants chosen purposively from production, agricultural, factory, and finance and procurement staff. We checked key figures against both sources, put everything in the same units (tons, hours, kilowatt-hours and naira), and compared monthly totals with the recorded totals.

Demand, unit costs, capacities, raw material availability and opening stock were taken from company records (Tables 2 and 3). Where the records did not provide labour-hour, machine-hour, electric power and storage parameters, realistic values consistent with the regular and overtime capacities were assumed so that the model could be solved; these are marked in Table 3 and should be replaced by plant records when they become available. Entries marked "Records" were read directly from SSCL production and cost records, and entries marked "Assumed" were set by the authors. To keep the two kinds of input apart, the assumed parameters are marked with a dagger (†) in Table 1 and are described as assumed wherever they are discussed in Section 4, and their influence on the results is tested in Section 4.5. Practitioners who wish to use the model directly should replace every assumed value with a measured plant value before relying on the plan.

3.2 General LP Formulation

A linear program chooses non-negative activity levels x_j to minimize a linear cost subject to linear resource limits:

$$Z = \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad i = 1, 2, \ldots, m \tag{2}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, n \tag{3}$$

where Z is the total cost, c_j the unit cost of activity j, a_{ij} the amount of resource i used per unit of activity j, and b_i the amount of resource i available. The method assumes proportionality, additivity, divisibility and certainty (Hillier & Lieberman, 2015).

3.3 Model Formulation

Each month $t = 1, \ldots, 6$ (January to June), management chooses regular-time production X_t^R, overtime production X_t^O and closing stock I_t, all in tons, so the model has 18 decision variables. Demand is known and must be met in full, so shortages and back-orders are not permitted. Sugar is treated as one aggregate product, sugar produced in a month can serve demand in that month, and all unit costs and resource coefficients are constant per ton. Table 1 defines the notation; parameters marked † are assumed values (Table 3).

Table 1: Notation of the production planning model

Symbol	Description	Unit
D_t	Demand for sugar in month t	tons
I_0	Opening stock at the start of January	tons
$X_t^R,\ X_t^O$	Regular-time and overtime production (decision variables)	tons
I_t	Closing stock at the end of month t (decision variable)	tons
$c_t^R,\ c_t^O$	Regular-time and overtime production cost per ton	₦/ton
h_t	Holding cost per ton per month	₦/ton/month
p_t	Electricity tariff	₦/kWh
$e_t^R,\ e_t^O$	Electric power per ton, regular time and overtime	kWh/ton
$E_t^R,\ E_t^O$	Electric power available, regular time and overtime	kWh
$R_t,\ O_t$	Regular-time and overtime production capacity	tons
$a_t,\ S_t$	Raw material per ton of sugar; raw material available	tons/ton; tons
$L_t^R,\ L_t^O,\ H_t^R,\ H_t^O$	Labour hours per ton; labour hours available (regular, overtime)	hours/ton; hours
$M_t^R,\ M_t^O,\ T_t^R,\ T_t^O$	Machine hours per ton; machine hours available (regular, overtime)	hours/ton; hours
K_t	Storage capacity	tons

Assumed where company records were not available (see Table 3).

Each ton of sugar carries its production cost and the cost of the electricity it consumes. The effective unit costs are

$$w_t^R = c_t^R + p_t e_t^R \tag{4}$$

$$w_t^O = c_t^O + p_t e_t^O \tag{5}$$

and the objective is to minimize the total of regular, overtime, electricity and holding costs over the six months:

$$Z = \sum_{t=1}^{6} (w_t^R X_t^R + w_t^O X_t^O + h_t I_t) \tag{6}$$

The model is subject to the following constraints for every month $t = 1, \ldots, 6$. The inventory balance states that closing stock equals opening stock plus production less demand:

$$I_t = I_{t-1} + X_t^R + X_t^O - D_t \tag{7}$$

Production in each time category cannot exceed its capacity:

$$X_t^R \leq R_t \tag{8}$$

$$X_t^O \leq O_t \tag{9}$$

The raw material used by all production cannot exceed the raw material available:

$$a_t(X_t^R + X_t^O) \leq S_t \tag{10}$$

Labour hours, machine hours and electric power (all assumed) are limited separately for regular time and overtime:

$$L_t^R X_t^R \leq H_t^R \tag{11}$$

$$L_t^O X_t^O \leq H_t^O \tag{12}$$

$$M_t^R X_t^R \leq T_t^R \tag{13}$$

$$M_t^O X_t^O \leq T_t^O \tag{14}$$

$$e_t^R X_t^R \leq E_t^R \tag{15}$$

$$e_t^O X_t^O \leq E_t^O \tag{16}$$

Closing stock cannot exceed the storage capacity, and all decision variables are non-negative:

$$I_t \leq K_t \tag{17}$$

$$X_t^R,\ X_t^O,\ I_t \geq 0 \tag{18}$$

Eqs. (7) to (17) give 11 constraint groups of six months each, so the model has 66 functional constraints in addition to the non-negativity conditions in Eq. (18).

3.4 Implementation and Analysis

We built the model in one Excel worksheet, with months in columns and separate blocks for the data, the decision cells, the cost formulas, and the left-hand side, right-hand side and slack of every constraint. In Solver, the total cost cell was set to minimum and the 18 decision cells were the changing cells. The inventory balance was entered as an equality and the other constraints as less-than-or-equal-to, unconstrained variables were made non-negative, and the Simplex LP method was selected (Fylstra *et al.*, 1998). We then used the Sensitivity Report to read off shadow prices, reduced costs and allowable ranges (Winston & Goldberg, 2004; Taha, 2017).

Table 2: Demand and raw material available, January to June (company records)

Item	Jan	Feb	Mar	Apr	May	Jun
Demand (tons)	3,350	3,590	2,980	3,100	3,290	3,460
Raw material available (tons)	32,091	31,801	32,035	32,226	32,319	32,409

Table 3: Parameters held constant across the six months

Parameter	Regular time	Overtime	Source
Production cost (₦/ton)	230,000	280,000	Records
Holding cost (₦/ton/month)	25,000	–	Records
Production capacity (tons/month)	2,700	1,300	Records
Raw material per ton of sugar (tons)	10	10	Records
Opening stock, January (tons)	2,000	–	Records
Labour per ton (hours)	0.60	0.30	Assumed
Labour available (hours/month)	1,650	400	Assumed
Machine time per ton (hours)	0.30	0.35	Assumed
Machine time available (hours/month)	900	500	Assumed
Electric power per ton (kWh)	120	135	Assumed
Electric power available (kWh/month)	330,000	180,000	Assumed
Electricity tariff (₦/kWh)	209.5	209.5	Records
Storage capacity (tons)	3,000	–	Assumed

Set at realistic values consistent with the regular (2,700 t) and overtime (1,300 t) capacities where company records did not provide them.

From Eqs. (4) and (5), the effective cost of a ton of regular-time sugar is 230,000 + 209.5 × 120 = ₦255,140, and that of an overtime ton is 280,000 + 209.5 × 135 = ₦308,282.50. Overtime therefore carries a premium of ₦53,142.50 per ton, while holding a ton for one month costs ₦25,000.

4. Result and Discussion

This section presents the results of solving the model in Excel Solver. Section 4.1 gives the optimal production plan and Section 4.2 its cost. Section 4.3 identifies the constraints that bind, and Sections 4.4 and 4.5 show how the solution responds to changes in the model's coefficients and in the assumed parameters. Section 4.6 turns the shadow prices into decision rules for plant managers, and Section 4.7 discusses what the results mean.

4.1 Optimal Production Plan

Solver found an optimal solution that satisfied every constraint, after 18 Simplex iterations. The least total cost over the six months is ₦4,678,873,844.50. Table 4 and Fig. 1 show the plan.

Table 4: Optimal production and inventory plan (tons)

Month	Demand	Opening stock	Regular production	Overtime production	Total production	Closing stock
Jan	3,350	2,000.0	2,674.6	0.0	2,674.6	1,324.6
Feb	3,590	1,324.6	2,700.0	0.0	2,700.0	434.6
Mar	2,980	434.6	2,700.0	0.0	2,700.0	154.6
Apr	3,100	154.6	2,700.0	522.6	3,222.6	277.2
May	3,290	277.2	2,700.0	531.9	3,231.9	219.1
Jun	3,460	219.1	2,700.0	540.9	3,240.9	0.0
Total	**19,770**	**2,000**	**16,174.6**	**1,595.4**	**17,770.0**	**0.0**

In the Total row, opening stock is the January opening stock and closing stock is the June closing stock.

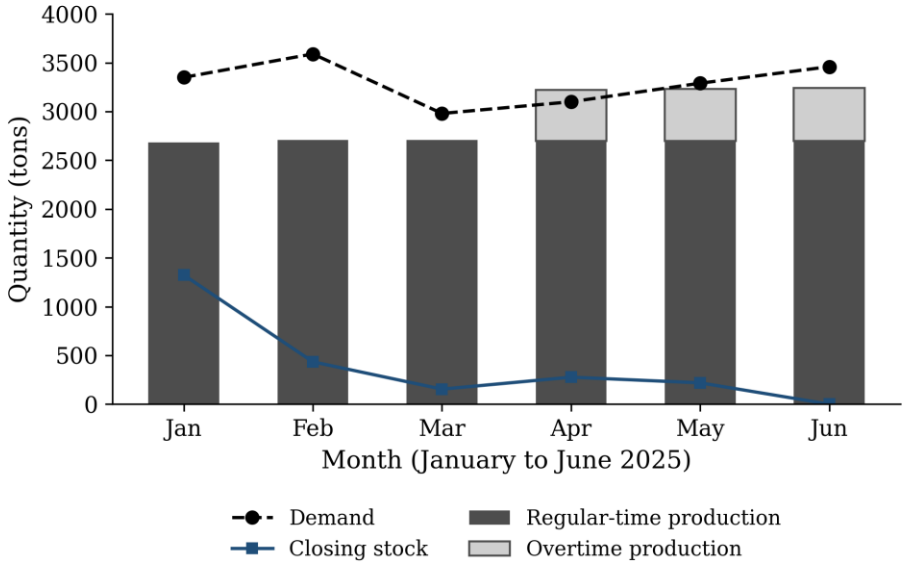


Figure. 1. Optimal production plan, demand and closing stock, January to June

Demand over the period totals 19,770 tons. The 2,000 tons of opening stock cover part of this, leaving 17,770 tons to be produced: 16,174.6 tons (91.0%) in regular time and 1,595.4 tons (9.0%) in overtime. Regular-time output sits at, or just below, the 2,700-ton limit in all six months. Overtime is switched on only from April to June, using roughly 40% to 42% of the 1,300 tons available.

Two features of the plan deserve comment. The first is January, where regular production (2,674.6 tons) is 25.4 tons below capacity. A ton made in January and held until April, when overtime first appears, would cost 3 × ₦25,000 = ₦75,000 to store, which is more than the ₦53,142.50 overtime premium. It is cheaper to leave that capacity idle and cover the April demand with overtime. The second is overtime in April to June, which comes from the raw material limit. At 10 tons of raw material per ton of sugar, the cane available allows only 3,222.6, 3,231.9 and 3,240.9 tons in those months. That is below demand in May (3,290 tons) and June (3,460 tons), so stock has to be carried forward. Stock builds to 277.2 tons at the end of April and 219.1 tons at the end of May, and is used up by the end of June.

4.2 Cost of the Optimal Plan

The minimum total cost of ₦4,678,873,844.50 is made up of regular-time production, overtime production, electricity and holding costs. Table 5 gives each component and its share of the total, which shows where the cost of the plan arises.

Table 5: Components of the minimum total cost

Cost component	Amount (₦)	Share of total (%)
Regular production cost	3,720,158,000.00	79.5
Overtime production cost	446,712,000.00	9.5
Electricity cost, regular time	406,629,444.00	8.7
Electricity cost, overtime	45,121,900.50	1.0
Holding cost	60,252,500.00	1.3
Total cost, Z	**4,678,873,844.50**	**100.0**

Regular production is the dominant cost at 79.5% of the total, followed by electricity (9.7%), overtime production (9.5%) and holding cost (1.3%). Holding cost is minor because stock stays low and is kept only briefly.

4.3 Binding Constraints

Only two kinds of constraint bind: regular-time capacity (February to June) and raw material (April to June). Everything else has slack. Overtime never exceeds 42% of its capacity, and storage peaks at 1,324.6 tons against 3,000 tons of space. Regular-time labour (1,620 of 1,650 assumed hours) and electric power (324,000 of 330,000 assumed kWh) come within about 2% of their limits, and regular machine time reaches 90% of its assumed limit. Because the labour, machine-time, electric power and storage limits were assumed rather than taken from records (Table 3), these margins are the least certain part of the results, and their influence is tested in Section 4.5.

4.4 Sensitivity Analysis

Sensitivity analysis shows how far the optimal plan and its cost respond to changes in the data, and standard texts treat it as part of any LP analysis (Winston & Goldberg, 2004; Taha, 2017). Recent optimization studies published in NIJOR also test their models in this way (Ahmed *et al.*, 2026; Onyenike *et al.*, 2026). Here the results are read from the Solver Sensitivity Report in three parts: the shadow prices of the binding constraints (Section 4.4.1), the marginal cost of demand (Section 4.4.2), and the cost coefficients and key relations (Section 4.4.3). Section 4.5 then tests the parameters that were assumed rather than taken from records.

4.4.1 Shadow prices of the binding constraints

A shadow price measures how total cost responds to one more unit of a constraint's right-hand side; Solver shows a negative value when the extra unit lowers cost. Table 6 gives the shadow prices of the binding constraints. Both binding constraints use data from company records.

Table 6: Shadow prices of the binding constraints

Constraint	Month	RHS	Shadow price (₦/unit)	Allowable increase	Allowable decrease
Regular capacity (tons)	Feb	2,700	-25,000.00	50	25.4
Regular capacity (tons)	Mar	2,700	-50,000.00	50	25.4
Regular capacity (tons)	Apr	2,700	-53,142.50	50	777.4
Regular capacity (tons)	May	2,700	-53,142.50	50	768.1
Regular capacity (tons)	Jun	2,700	-53,142.50	50	759.1
Raw material (tons)	Apr	32,226	-2,185.75	1,546	254
Raw material (tons)	May	32,319	-4,685.75	1,546	254
Raw material (tons)	Jun	32,409	-7,185.75	1,546	254

One more ton of regular capacity lowers total cost by ₦53,142.50 in April, May and June, because it replaces an overtime ton and the saving is the overtime premium. In February and March the saving is ₦25,000 and ₦50,000, because the extra regular ton displaces January output that would have been held for one or two months. These prices hold for increases of up to 50 tons, so 50 extra tons in April would save 50 × ₦53,142.50 = ₦2,657,125. For raw material, one more ton saves ₦2,185.75 in April, ₦4,685.75 in May and ₦7,185.75 in June, for increases up to about 1,546 tons. An extra 1,000 tons in May would save ₦4,685,750.

4.4.2 Marginal cost of demand

The shadow prices of the inventory-balance equations give the marginal cost of supplying one more ton of demand in each month (Table 7 and Fig. 2).

Table 7: Marginal cost of meeting additional demand

Month	Marginal cost (₦/ton)	Allowable increase (tons)	Allowable decrease (tons)
Jan	255,140	25.4	2,674.6
Feb	280,140	25.4	1,324.6
Mar	305,140	25.4	434.6
Apr	330,140	25.4	154.6
May	355,140	25.4	154.6
Jun	380,140	25.4	154.6

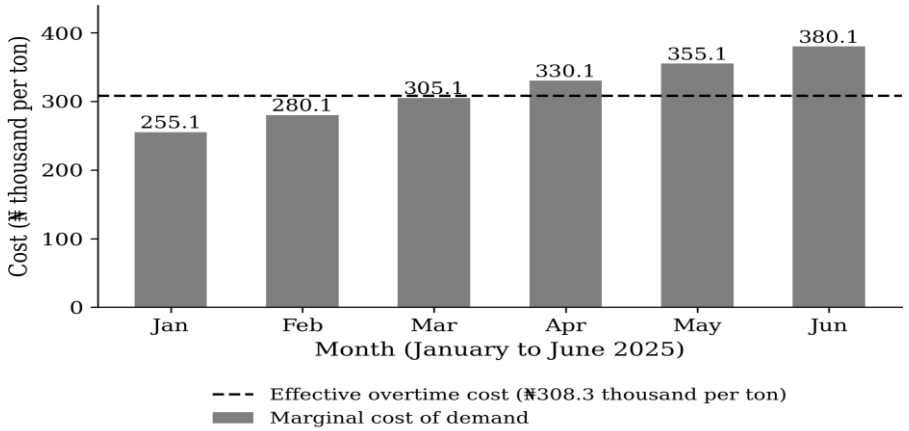


Figure. 2. Marginal cost of demand by month against the effective overtime cost per ton

The marginal cost rises by ₦25,000 each month, from ₦255,140 in January to ₦380,140 in June. The least costly source of an extra ton in any month is January regular-time production, where 25.4 tons of capacity are idle, kept in stock until required. The marginal cost of demand in month t is therefore

$$\pi_t = w^R + h(t - 1) \tag{19}$$

which holds as long as the extra demand does not exceed the 25.4 tons of unused January capacity. Eq. (19) can serve as a rule for pricing and for accepting extra orders, since it gives the marginal cost of each month's additional order.

4.4.3 Cost coefficients and key relations

Overtime would be chosen in January, February or March only if its effective unit cost dropped by ₦53,142.50, ₦28,142.50 or ₦3,142.50 respectively. The March figure is small, which means overtime in March and carrying January output for two months are nearly equal in cost. For April, the plan stays optimal for any effective overtime cost from ₦255,140 to ₦330,140 per ton, a wide band around today's ₦308,282.50.

The results can be summarized by two further relations. The break-even holding period, the number of months for which stock can be carried before overtime becomes cheaper, is

$$\tau^* = \frac{w^O - w^R}{h} = \frac{53{,}142.50}{25{,}000} \approx 2.1\ months \tag{20}$$

and the saving from one extra ton of raw material in an overtime month t (April to June) is

$$\lambda_t = \frac{\pi_t - w^O}{a_t} \tag{21}$$

Eq. (21) reproduces the Solver values: (330,140 − 308,282.50)/10 = ₦2,185.75 in April, (355,140 − 308,282.50)/10 = ₦4,685.75 in May and (380,140 − 308,282.50)/10 = ₦7,185.75 in June. An extra ton of raw material permits an extra tenth of a ton of sugar, which is produced on overtime at the cost w^O and replaces a ton that would otherwise be supplied at the marginal cost π_t.

4.5 Sensitivity to the Assumed Parameters

Because the labour-hour, machine-hour, electric power and storage parameters were assumed (Table 3), we tested how far the optimal solution depends on them. We changed each assumed parameter by 10% and by 20% in the adverse direction (more use per ton, or a lower limit), one at a time, re-solved the model each time, and recorded the change in total cost and the largest adverse change that leaves the production plan unchanged (Table 8).

Table 8: Sensitivity of the optimal plan to the assumed parameters (one parameter changed at a time)

Assumed parameter	Adverse change	Plan unchanged up to	Cost change at 10%	Cost change at 20%
Labour hours per ton	Increase	1.9%	+1.04%	+2.32%
Labour hours available	Decrease	1.9%	+1.17%	+2.89%
Machine hours per ton	Increase	11.2%	0.00%	+1.04%
Machine hours available	Decrease	10.1%	0.00%	+1.64%
Electric power per ton	Increase	1.9%	+2.01%	+4.28%
Electric power available	Decrease	1.9%	+1.17%	+2.89%
Storage capacity	Decrease	55.8%	0.00%	0.00%

Cost change is relative to the base total cost of ₦4,678,873,844.50. For electric power per ton, the change includes the direct effect on electricity cost.

The plan is insensitive to the storage and machine-time assumptions: stock peaks at 1,324.6 tons, which is 44% of the assumed storage capacity, and machine time can be about 10% worse than assumed without any chl2ange. The plan is sensitive to the regular-time labour and electric power limits, because planned use is within 2% of both limits (Section 4.3). An adverse change larger than 1.9% makes these limits bind, regular-time output falls below 2,700 tons, and overtime starts earlier: in March when the change is 10% and in February when it is 20%. Even so, no single adverse change of 20% raises total cost by more than 4.3%, and a simultaneous adverse change of 10% in all six labour, machine-time and electric power parameters raises it by 3.6%.

Electricity tariffs also change over time, so we varied the tariff as well. A change of 10% in either direction changes the total cost by 0.97% (about ₦45.2 million) and a change of 30% by 2.9%, with the production plan unchanged. These tests do not replace measured plant values. The regular-time labour and electric power limits should be verified first, because they have the largest influence on the plan.

4.6 Using the Results in Practice

Plant managers can use the shadow prices and allowable ranges directly when raw material supply or demand changes unexpectedly, without rebuilding the model. Table 9 gives decision rules, with examples that were checked by re-solving the model.

Table 9: Decision rules for unexpected changes

Situation	Rule	Example
Cane delivery in month t is short by up to 254 tons	Extra cost = shortfall × λ_t (₦2,185.75 April, ₦4,685.75 May, ₦7,185.75 June). No re-solve is needed.	100 tons short: +₦218,575 (April), +₦468,575 (May), +₦718,575 (June)
Shortfall is larger than 254 tons	The shadow price no longer applies exactly. Update the data and re-solve.	300 tons short in April: cost rises by ₦670,180.50, not ₦655,725, and overtime starts in March
Extra cane is offered (up to 1,546 tons)	Accept it only if its delivered price per ton is below λ_t; the saving is λ_t per ton.	1,000 extra tons in April save ₦2,185,750
A customer asks for extra sugar in month t	Quote at least π_t (₦255,140 in January to ₦380,140 in June per ton) for up to 25.4 tons. Beyond this, re-solve.	An order for 20 tons in June should be priced at no less than ₦380,140 per ton
Cane supply falls in every month	Re-solve with the new supply.	A 5% fall raises cost by 0.58% (about ₦27.1 million) and starts overtime in February; a fall of 8% makes full demand impossible to meet
Tariff, overtime cost or holding cost changes	Recompute $w^O - w^R$ and the break-even period in Eq. (20).	A 10% tariff change changes total cost by 0.97% without changing the plan

Each month the planning office can update the demand, raw material and tariff cells, re-solve the model, and compare the new shadow prices with these rules. The results of a 5% fall in cane supply, and the infeasibility at 8%, show that raw material is the first constraint to watch.

4.7 Discussion

The results show how three cost drivers interact: the overtime premium of ₦53,142.50 per ton, the holding cost of ₦25,000 per ton per month, and the raw material limit. The overtime premium divided by the monthly holding cost gives a break-even holding period of about 2.1 months (Eq. (20)). Storing a regular-time ton for up to two months therefore costs less than making it on overtime, while storing it for three months or longer costs more. This explains the pattern of the plan: regular time is exhausted from February onward, stock is held only briefly, and overtime enters in April to June once earlier stock can no longer cover demand economically. It is the same regular-time, overtime and inventory trade-off that characterizes the classic aggregate planning models (Bowman, 1956; Hanssmann & Hess, 1960; Holt *et al.*, 1960).

A sl2econd finding is that raw material limits output in April to June, so extra raw material and extra regular capacity both reduce cost, most of all in the later months. Shadow prices reveal where an added unit of capacity or raw material pays off most, marginal costs price an extra order month by month, and the allowable ranges mark how far costs may drift before the plan needs revisiting. None of this can be read from a single cost calculation. The tests in Section 4.5 add that the structure of the plan depends mainly on the assumed regular-time labour and electric power limits, which is why these two values should be confirmed with plant records.

5. Conclusion

We formulated and solved a six-month LP model for least-cost production planning at SSCL, Numan, with 18 decision variables and 66 constraints, counting electric power as a cost. The minimum total cost is ₦4,678,873,844.50. In the optimal plan, 16,174.6 tons come from regular time and 1,595.4 tons from overtime, which is confined to April to June, and the stock is fully used by the end of the horizon. Regular capacity and raw material are the binding constraints, valued at ₦25,000 to ₦53,142.50 per ton of regular capacity and ₦2,185.75 to ₦7,185.75 per ton of raw material. LP with Excel Solver is a practical tool for monthly production planning in a Nigerian sugar plant. The plan is robust to the assumed storage and machine-time values but sensitive to the assumed regular-time labour and electric power limits, which should be verified first.

For management, four practical points follow. First, SSCL should run regular time at capacity from February to June and avoid overtime in the first three months. Second, overtime belongs in April to June, and any extra cane should be sought for those months, where a ton is worth up to ₦7,185.75. Third, the shadow prices give a ready value for any plan to expand regular capacity: a ton is worth ₦53,142.50 a month in April to June. Fourth, stock should not be built more than about two months ahead of the demand it serves, and Eq. (19) can guide pricing and the acceptance of extra orders.

The study has limits. The model is deterministic, so demand, costs and resource availability are treated as certain. Sugar is one aggregate product, costs are linear, shortages and back-orders are not allowed, and the horizon is six months. Several resource parameters were assumed where records were not available, so the plan should be rerun with plant data.

A practical roadmap for later phases follows from the sensitivity results. First, scenario analysis: build a small set of scenarios for cane supply (for example −5%, base and +5%) and for the electricity tariff (−10%, base and +10%) from the plant's records, and re-solve the model for each, as was done here for single changes. Second, a two-stage stochastic program, in which regular-time production and stock are decided first, and overtime and any shortage are recourse decisions taken once cane supply and tariffs are known, minimizing expected cost over the scenarios. Third, if probabilities are hard to estimate, a robust version in which cane supply may vary within a stated band and the plan must stay feasible, with the band set from the plant's own history; the infeasibility found for an 8% fall in cane shows why this matters. Fourth, a rolling horizon in which the plan is re-solved every month with updated data. Surveys of planning under uncertainty and fuzzy goal programming for a sugar mill (Jamalnia *et al.*, 2019; da Silva & Marins, 2014) give starting points for these steps, and Onyenike *et al.* (2026) show how fuzzy demand can be handled in an inventory model. Other extensions include back-orders and lost-sales costs, hiring and layoff decisions, by-products of sugar production, and multiple objectives such as the cost, rejected-unit and delivery criteria combined by Ahmed *et al.* (2026).

Author Contributions

Yahaya Garba: Conceptualization, Data curation, Methodology, Formal analysis, Software, Writing - original draft. Ayuba Modu Yami & Afolabi Oludele Raphael: Supervision, Validation, Writing – review and editing.

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Data Availability

The input data are given in Tables 2 and 3, and the Excel Solver workbook is available from the corresponding author on reasonable request.

Conflicts of Interest

The authors declare no conflict of interest.

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