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A graph analysis of topological structure in the bus rapid transit transport network of Lagos state

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ABSTRACT

The transportation system in Nigeria's city of Lagos is a complex network characterized by rapid urbanization, significant congestion, and a mix of formal and informal modes of transportation. This paper proposes a graph-theoretic framework for structurally and functionally modelling and analyzing the Bus Rapid Transit terminal transportation system in Lagos using a vertex deletion algorithm. The system is represented as an undirected graph representing the Bus Rapid Transit Lite system. The analysis yields a hub-and-spoke in the road network, with susceptibility to interference at nodes of high centrality such as Surulere. The Bus Rapid Transit terminal, while successful along its route, lacks strong integration with the overall network. A graph-theoretic model offers a framework for guiding targeted investments to make Lagos's transport system more efficient, inclusive, and sustainable.

1. Introduction

Urban transport networks are the lifelines of modern cities, shaping economic productivity, social equity, and environmental sustainability (Nawazish *et al.*, 2024; Yedla, 2015). The efficient movement of people and goods remains a persistent challenge for planners, particularly in rapidly expanding megacities of the Global South, where infrastructure growth often struggles to match population surges (Adeyemi & Olaniyi, 2024). Lagos, Nigeria, exemplifies this dynamic, with a population estimated at over 20 million and a transport system that is both congested and fragmented. Gaining a structural understanding of this network and its vulnerabilities is, therefore, a vital foundation for informed urban planning and policy development.

The transport ecosystem in Lagos is uniquely complex. It is dominated by an extensive informal sector, which operates alongside formalized systems such as the Bus Rapid Transit (BRT) and a nascent urban rail project (Oyalowo *et al.*, 2024). This diversity creates a heterogeneous system with emergent properties that are difficult to capture through conventional transport modelling techniques. Among these modes, however, BRT has emerged as one of the most efficient commuting options in Lagos State. Designed as a cost-effective, bus-based system, BRT leverages dedicated lanes, high-frequency

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operations, and simplified boarding to deliver rapid, reliable, and affordable mobility. The system combines the efficiency of rail transit with the flexibility of buses, yielding multiple benefits such as reduced travel times, lower congestion levels, improved air quality, and enhanced accessibility (Olufemi, 2008; Sekgale, 2020).

To understand such complexity, graph theory offers a powerful analytical lens. As a branch of discrete mathematics, graph theory models intersections as nodes (vertices) and the connecting roads as links (edges) (Newman, 2010). This abstraction enables researchers to move beyond descriptive approaches by applying quantitative measures to evaluate network topology, performance, and resilience. The methodology has been successfully employed in diverse contexts, from analyzing the subway systems of London and Tokyo to assessing airline traffic networks across continents (Derrible & Kennedy, 2010).

Despite its promise, research on Lagos's transport system has so far concentrated on sociological surveys, traffic counts, and policy-level assessments, with limited focus on systemic structural analysis. While some studies have noted the significance of network structures, a comprehensive graph-theoretic investigation that integrates the BRT terminal's location is still missing. Addressing this gap, Section 2 introduces relevant graph terminologies aligned with the study's objectives. Section 3 develops the BRT terminal network graphs derived from the Lagos transport map, followed by recommendations on strategically optimal local government areas for siting new BRT terminals.

2. Graph terminologies

The adjacency matrix, $A(G) = [a_{i,j}]$, according to Babarinsa & Hailiza, (2019) is a square matrix indexed by the vertices $\{v_1, v_2 \dots v_n\}$, denoted as given in Eq. (1)

$$a_{i,j} = \begin{cases} 1 & \text{if } v_i, v_j \text{ is an edge of } G \\ 0 & \text{otherwise .} \end{cases} \tag{1}$$

The adjacency matrix is useful not only in constructing the graph itself but also in determining the connectivity of a graph and obtaining the spectrum and energy of the graph, see (Babarinsa, 2022, 2026; Babarinsa & Ngule, 2025; Kekere *et al.*, 2026). A simple (undirected) graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ consists of a set of vertices V and a set of undirected edges E without loops (Diestel, 2025). A loop is an edge that links a vertex to itself. A pendant vertex is a vertex that joins exactly one other vertex, while an isolated vertex has no edge (Vasudev, 2006).

A new graph (subgraph) can be formed when an edge or vertex is deleted. An edge deletion is the removal of an edge e between two vertices of a graph G, denoted as $G - e$ (Lauri, 1993). This removal only affects the removed edge. Nevertheless, a vertex deletion is the removal of a vertex v from a graph G, along with all edges incident to v (Merris, 2011). The resulting graph is often denoted $G - v$. Vertex deletion provides a straightforward method for studying the structural properties of graphs and identifies vulnerable points in networks, especially transport networks. Vertex deletion helps reduce the complexity of large or complex graphs for easier analysis. It reveals critical nodes in networks by studying how deleting certain vertices affects connectivity.

3. BRT terminal network graph

The study area focused on the BRT terminals (mainland and island corridors) of Lagos state. Network data was extracted and digitized from OpenStreetMap (OSM) using QGIS software on map of Lagos state. The network was simplified to major nodes (terminals) and links (road links to terminals). Two complementary graph models were constructed, which are the map graph of Lagos state and the network graph of BRT terminals (before and after vertex deletion).

3.1 Lagos state map graph

The map graph of Lagos State can be achieved by considering the map of Lagos State in Fig. 1.

Fig.1. Map of Lagos State

Lagos State consists of 20 LGAs, and they are represented and denoted to form an adjacency matrix in Table 1. To begin with, each LGA represents a node, while the border between two LGAs represents a link. Representing the node and links forms the map graph of Lagos state, see Fig. 2.

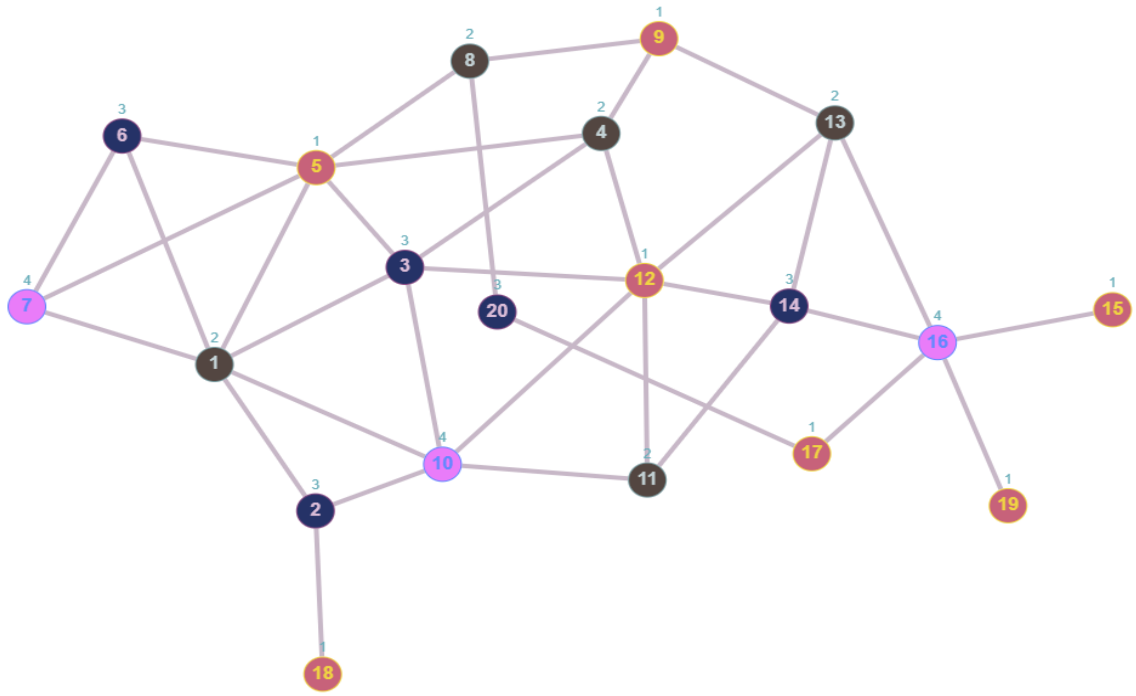

Fig. 2. Map graph of Lagos State

Where L_i's are defined each according to the LGAs as:

$L_1 =$ Alimosho; $L_2 =$ Ojo; $L_3 =$ Oshodi-Isolo; $L_4 =$ Mushin; $L_5 =$ Ikeja; $L_6 =$ Agege; $L_7 =$ Ifako-Ijaye; $L_8 =$ Kosofe; $L_9 =$ Shomolu; $L_{10} =$ Amuwo Odofin; $L_{11} =$ Ajeromi-Ifelodun; $L_{12} =$ Surulere; $L_{13} =$ Lagos mainland; $L_{14} =$ Apapa; $L_{15} =$ Lagos Island; $L_{16} =$ Eti-Osa; $L_{17} =$ Epe ; $L_{18} =$ Badagry; $L_{19} =$ Beju Lekki; $L_{20} =$ Ikorodu.

Table 1: Adjacency matrix of Lagos State

	L_1	L_2	L_3	L_4	L_6	L_6	L_7	L_s	L	L_{10}	L_{11}	L_{12}	L_{13}	L_{14}	L_{15}	L_{16}	L_{17}	L_{18}	L_{19}	L_{20}
L_1	0	1	1	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0
L_2	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
L_3	1	0	0	1	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
L_4	0	0	1	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
L_5	1	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
L_6	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
L_7	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L_s	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
L_9	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
L_{10}	1	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
L_{11}	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0
L_{12}	0	0	1	1	0	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0
L_{13}	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	0	0
L_{14}	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	0
L_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
L_{16}	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	1	0
L_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
L_{18}	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
L_{20}	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0

The map graph, in Fig. 2, indicates structural deficits where LGAs like Lagos Island graph ($L_{15} =$ Lagos Island), Badagry graph ($L_{18} =$ Badagry), and Ibeju-Lekki graph ($L_{19} =$ Beju Lekki) are pendant vertices due to their peripheral accessibility, locking them in the network. Although developing BRT terminals in these areas can enhance local employment and accessibility, it would have little systemic resilience due to their low centrality. The current eight key BRT terminals (Oshodi, Ikeja, Oyingbo, Agege, Ajah, Abule Egba, Ikorodu, and TBS) are a scale-free network that efficiently directs demand but also creates bottlenecks and single points of failure, such as the isolated Ikorodu node and the path-like structure of Eti-Osa and Lagos Island, that make cascading disruption more likely.

3.2 Vertex deletion model for BRT terminal

This section uses the minimum vertex cover via vertex deletion by deleting vertices that are part of the map graph until no edges remain, thereby optimizing network properties, see Algorithm 1.

```
class Graph:
    def __init__(self):
        self.adj = {}
    def add_vertex(self, v):
        if v not in self.adj:
            self.adj[v] = []
    def add_edge(self, u, v):
        self.adj[u].append(v)
        self.adj[v].append(u)
    def delete_vertex(self, v):
        if v not in self.adj:
            return
        for neighbor in self.adj[v]:
            self.adj[neighbor].remove(v)
        del self.adj[v]
    def display(self):
        for v in self.adj:
            print(f"{v} -> {self.adj[v]}")
```

Algorithm 1. Vertex deletion algorithm for Lagos map graph.

3.2.1 Lagos State BRT terminals

There are 8 terminals in Lagos state. These terminals connect all other terminals through the major route of the BRT. The terminals are: Oshodi terminal, Ikeja terminal, Oyingbo terminal, Agege terminal, Ajah terminal, Abule Egba terminal, Ikorodu terminal, and TBS terminal. These terminals are located in different local government areas, as shown in Table 2.

Table 2

BRT terminals and their LGAs

Terminals	LGA
Oshodi terminal	L_3 = Oshodi-Isolo
Ikeja terminal	L_5 = Ikeja
Oyingbo terminal	L_{13} = Lagos mainland
Agege terminal	L_6 = Agege
Ajah terminal	L_{16} = Eti-Osa
Abule Egba terminal	L_1 = Alimosho
Ikorodu terminal	L_{20} = Ikorodu
TBS terminal	L_{15} = Lagos Island

Map graph of Lagos State representing the local government area with their terminals is depicted in Fig. 3, and without BRT in Fig. 4.

Fig. 3. Map graph of Lagos State with BRT terminals.

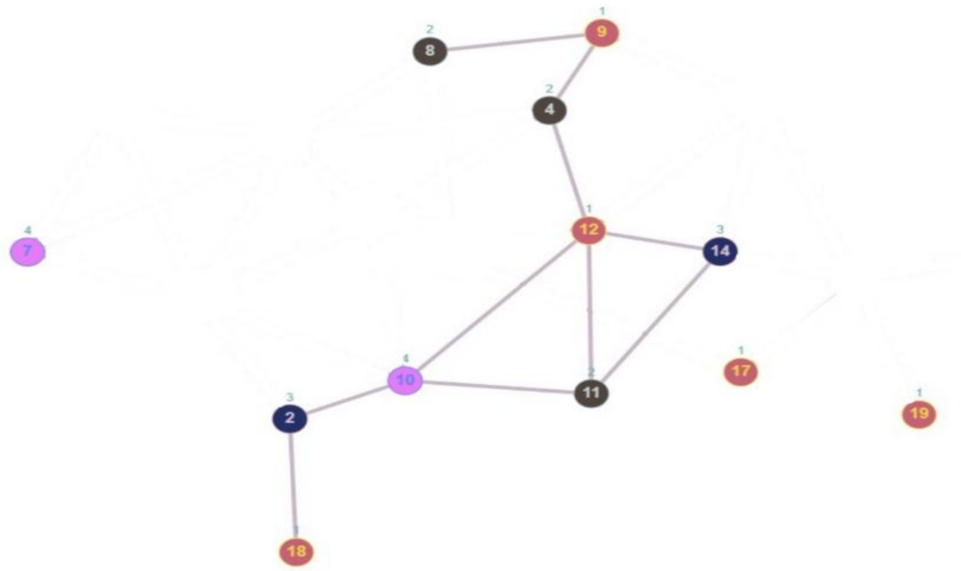

Fig. 4. Map graph of Lagos State without BRT terminals.

In Fig. 3, it is obvious that L_{13} = Lagos mainland, L_{15} = Lagos Island and L_{16} = Eti-Osa are path graphs while L_{20} = Ikorodu is an isolated node. This implies that centrality was not considered when planning to establish BRT terminals. However, Fig. 4. finds Surulere (L_{12} = Surulere) to be a strategic point for a second terminal, opening up four other LGAs ((L_4 =Mushin, L_{10} =Amuwo Odofin, L_{11} =Ajeromi-Ifelodun and L_{14} =Apapa)) to access and reducing travel distances while offloading pressure from over-saturated hubs, thus enhancing equity for inner-city commuters. Similarly, a terminal in Murtala Muhammed International Airport in Ikeja LGA (L_5 = Ikeja) would offer a significant multimodal interchange, raising Lagos's global accessibility as well as economic competitiveness. Apart from physical expansion, the vulnerability of the network due to its centralized convergence of flows necessitates operational upgrades like dynamic scheduling, feeder bus networks, and railway integration to dissipate demand and build resilience. These interventions have the potential to reduce reliance on unplanned transport and make it more accessible, secure, and sustainable, and correct the systemic weaknesses of isolation, congestion, and vulnerability that the graph-theoretic analysis shows.

Table 3.
Degree centrality and betweenness centrality of 20 LGAs

LGA	Degree	Degree Centrality	Betweenness Centrality
Alimosho	5	0.263	0.084
Ojo	2	0.105	0.010
Oshodi-Isolo	4	0.211	0.096
Mushin	3	0.158	0.052
Ikeja	6	0.316	0.142
Agege	2	0.105	0.008
Ifako-Ijaye	2	0.105	0.006
Kosofe	2	0.105	0.015
Shomolu	2	0.105	0.018
Amuwo-Odofin	4	0.211	0.061
Ajeromi-Ifelodun	2	0.105	0.020
Surulere	4	0.211	0.118
Lagos Mainland	3	0.158	0.067
Apapa	2	0.105	0.023
Lagos Island	1	0.053	0.000
Eti-Osa	4	0.211	0.074
Epe	2	0.105	0.012
Badagry	1	0.053	0.000
Ibeju-Lekki	1	0.053	0.000

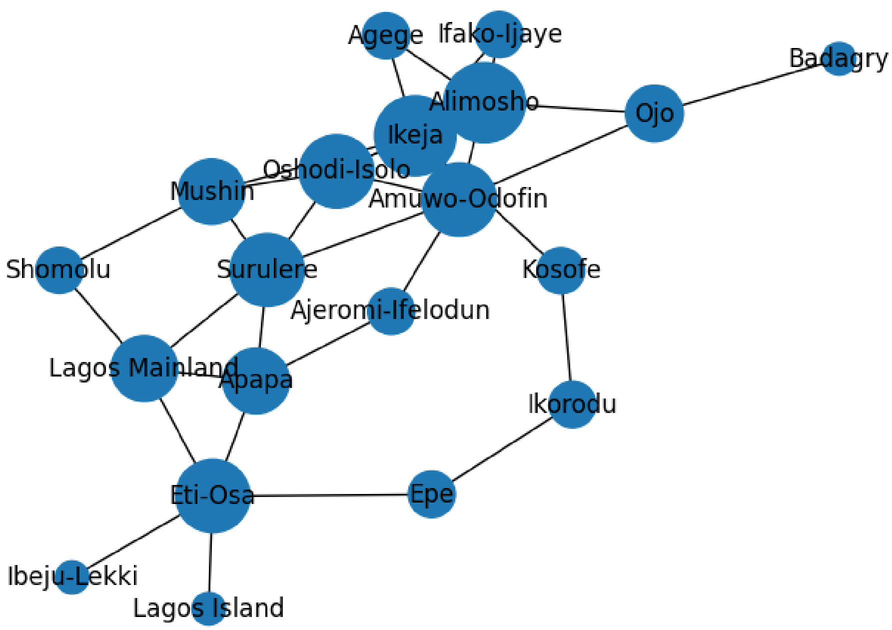

Fig.5. Degree centrality of Lagos State.

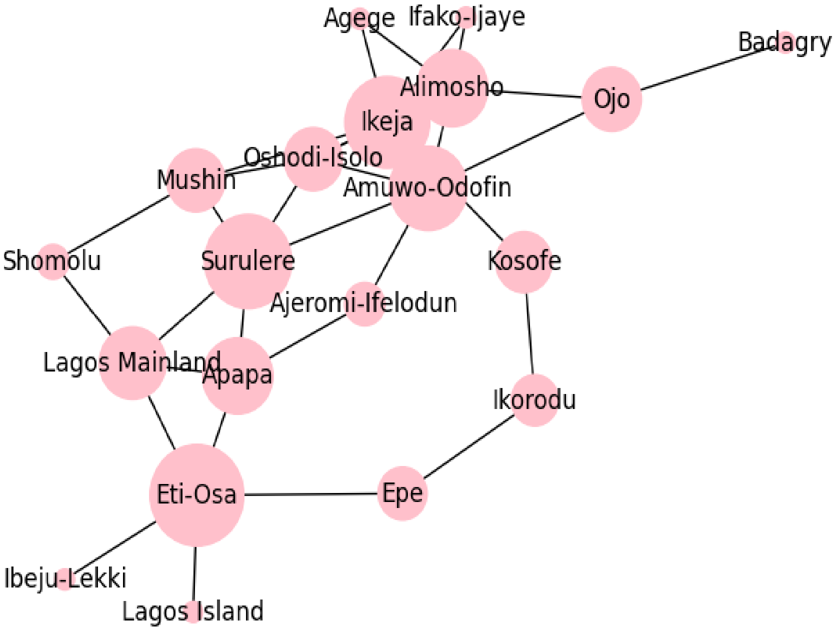

Fig.6. Betweenness centrality of Lagos State.

Degree centrality measures exposure and local interaction strength with node size reflects how many direct borders each LGA has. From Table 3, Ikeja has the highest degree and betweenness, while Surulere has the strongest intermediary control, followed by Oshodi-Isolo and Alimosho. In other words, Ikeja connects multiple clusters and lies on many shortest paths. Surulere is not the highest-degree node, but its betweenness is very high. Since Lagos Island, Badagry, and Ibeju-Lekki, see Fig. 5, have degree 1, they are peripheral spokes and are pendant vertices. Their vertex deletion does not collapse the network, but they depend heavily on their single neighbour. Ikorodu has degree 2, but not centrally embedded. In Fig. 6, Eti-Osa has an important role as a gateway between the island and mainland structures. Meanwhile, Agege and Ifako-Ijaye shrink significantly in the betweenness graph. They may have several neighbours, but they do not control inter-cluster traffic. They are locally active but globally replaceable.

4. Conclusion

The graph-theoretic analysis shows that Lagos's BRT network, though vital for mobility, is highly centralized and vulnerable to disruptions. Strategic expansion, particularly adding a terminal in Surulere to improve local connectivity and another at Ikeja Airport to integrate air and road transport, would ease bottlenecks, strengthen resilience, and enhance equity. In all, the study successfully demonstrates the significant utility of graph-theoretic concepts in diagnosing the BRT terminals. This model offers a framework for guiding targeted investments that can make Lagos's transport system more efficient, inclusive, and sustainable.

Author Contributions

Conceptualisation, B.O. and E.G; methodology, B.O, D.E. and E.G.; software, E.G.; validation, D.E.; resources, E.G.; data curation, E.G.; and D.E.; writing—original draft preparation, B.O.; writing—review and editing, B.O.; visualisation, E.G. and D.E.; supervision, B.O.; project administration, B.O. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest
The authors declare that they have no conflicting financial interests or personal connections that might lead to biases or otherwise affect the research described in this work. The authors declare that there are no conflicting interests that could compromise the paper's validity, objectivity, or integrity.

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References
1. Adeyemi, A., & Olaniyi, T. (2024). Sustainable planning and inclusive cities in the global south: A systemic review of literature: A case for Nigeria. *International Journal of Innovative Business Strategies*.
2. Babarinsa, O. (2022). Graph theory: A lost component for development in Nigeria. *Journal of the Nigerian Society of Physical Sciences, 4*(3), 844-851. doi:10.46481/jnsps.2022.874
3. Babarinsa, O. (2026). Strategic security zoning via chromatic graph theory: an optimized patrol allocation for nigeria's geopolitical region. *African Journal of Mathematics and Statistics Studies, 9*(1), 55-71.
4. Babarinsa, O., & Hailiza, K. (2019). Mixed energy of a mixed hourglass graph. *Communications in Mathematics and Applications, 10*(1), 45-53.
5. Babarinsa, O., & Ngule, S. (2025). Patriarchy Family Tree Graph from Tiv to Kumator (Seember). *International Journal of Development Mathematics, 2*(4), 137 - 147. doi:https://doi.org/10.62054/ijdm/0204.09
6. Derrible, S., & Kennedy, C. (2010). The complexity and robustness of metro networks. *Physica A: Statistical Mechanics and its Applications, 389*(17), 3678-3691.
7. Diestel, R. (2025). *Graph theory* (Vol. 173): Springer Nature.
8. Kekere, S. Y., Babarinsa, O., & Johnson, F. (2026). Graph topological indices and energies of a mixed hourglass graph. *African Journal of Mathematics and Statistics Studies, 9*(1), 101-107. doi: https://doi.org/10.52589/AJMSS-JVC7MCCV
9. Lauri, J. (1993). Endvertex-deleted subgraphs. *Ars Combinatoria, 36*, 171-184.
10. Merris, R. (2011). *Graph theory*: John Wiley & Sons.
11. Nawazish Ali, S. M., Ahmadi, S., Amani, A. M., & Jalili, M. (2024). Urban integrated sustainable transportation networks. *Interconnected Modern Multi-Energy Networks and Intelligent Transportation Systems: Towards a Green Economy and Sustainable Development*, 59-91.
12. Newman, S. (2010). *Networks: An Introduction*. USA: Oxford university press.
13. Olufemi, O. A. (2008). Public transport innovation: the impact of BRT on passenger's movement in Lagos metropolitan area of Nigeria. *Pakistan Journal of Social Sciences, 5*(8), 845-852.
14. Oyalowo, B., Udoma-Ejorh, O., Muraina, O., Lawanson, T., & Nubi, T. (2024). Urban Infrastructure and Transit-Oriented Development in Lagos, Nigeria *Transit Oriented Development in West African Cities* (pp. 281-301): Springer.
15. Sekgale, M. D. (2020). *Assessing the effectiveness of the operational strategy of Bus Rapid Transit (BRT) in Gauteng*: University of Johannesburg (South Africa).
16. Vasudev, C. (2006). *Graph theory with applications*: New Age International.
17. Yedla, S. (2015). Urban transportation and the environment. *Urban Transportation and the Environment: Issues, Alternatives and Policy Analysis; Springer: New York, NY, USA*.