Reliability Evaluation and Optimization of Solar Based Ammonia Urea Production System: A Study on Lethal and Nonlethal Failures with Coverage Factor Considerations

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Received: **September, 2025**

Accepted for publication: **November, 2025**

Published: **December, 2025**

Abstract

This study presents a comprehensive reliability evaluation and optimization framework for a solarbased ammonia-urea production system designed for Kano State, Nigeria, emphasizing the effects of lethal and nonlethal failures under varying coverage factor conditions. The proposed system integrates solar photovoltaic (PV) generation with electrolyzers, ammonia synthesis, and urea production units to promote sustainable fertilizer manufacturing suited to the climatic and energy conditions of Kano State. Reliability modeling was conducted using Reliability Block Diagrams (RBD) and Markov analysis to assess system performance under different failure and repair scenarios. Lethal failures, leading to total system shutdown, and nonlethal failures, causing partial performance loss, were analyzed to determine their impact on system availability, reliability, sensitivity and Cost (CARS). The coverage factor was incorporated to capture the influence of fault detection and recovery efficiency. Optimization studies focusing on redundancy allocation and improved fault coverage were performed to enhance system resilience. Results indicate that coverage factor placement significantly decrease system reliability, availability, or cost thereby making the solar-based ammonia-urea production system unviable and unsustainable solution for Kano State's agricultural and energy needs.

Keywords: Solar energy systems, Reliability assessment, Coverage factor, CARS approach, Kano Metropolis, Optimization.

1. Introduction

The continuous rise in global population and the subsequent increase in food demand have significantly intensified the need for large-scale fertilizer production. Ammonia and urea remain the most widely used nitrogen-based fertilizers worldwide, playing a crucial role in enhancing agricultural productivity. Traditionally, these fertilizers are produced through energy-intensive processes that depend heavily on fossil fuels, particularly natural gas. Such dependency not only leads to high production costs but also contributes substantially to greenhouse gas emissions, posing environmental and sustainability challenges. Consequently, there has been a growing interest in integrating renewable energy sources into fertilizer production systems to achieve cleaner, more sustainable operations.

Among various renewable options, solar energy stands out as a viable and abundant source, especially in regions with high solar irradiance such as Kano State, Nigeria. Located in the Sahelian region, Kano State experiences an average daily solar radiation intensity of about 5–7 kWh/m^2, making it one of the most promising areas for solar energy exploitation in West Africa. The state's agricultural economy further emphasizes the need for a sustainable local fertilizer production system that reduces reliance on imported fertilizers and ensures steady availability for farmers. Thus, the development of a solar-based ammonia-urea production system in Kano State presents both economic and environmental advantages.

However, the integration of solar energy into ammonia-urea production introduces several technical and operational challenges, particularly regarding system reliability. Unlike conventional plants powered by stable energy sources, solar-based systems are subject to fluctuations in energy availability due to diurnal and seasonal variations. Additionally, the system comprises multiple interdependent subsystems such as solar power generation units, hydrogen production through electrolysis, nitrogen extraction modules, and synthesis reactors each of which can experience different types of failures. These failures can broadly be categorized as lethal (causing complete system shutdown) or nonlethal (causing partial degradation of performance without total failure).

To accurately assess and enhance system dependability, this study incorporates the concept of a coverage factor, which represents the probability that the system can successfully detect, isolate, and recover from a component failure. Considering this factor provides a more realistic measure of system performance under failure conditions and helps in identifying weak points that significantly influence reliability.

Therefore, this research focuses on the reliability evaluation and optimization of a solar-based ammonia–urea production system designed for Kano State. Using reliability modeling techniques such as fault tree analysis, Markov modeling, or reliability block diagrams the study aims to quantify the impact of various failure modes, both lethal and nonlethal, on system availability and performance. Furthermore, optimization strategies are applied to improve system configuration, fault tolerance, and operational continuity under local environmental conditions.

The findings from this study are expected to contribute valuable insights into the design and operation of renewable-based fertilizer plants suitable for sub-Saharan Africa. By improving the reliability and efficiency of solar-driven ammonia–urea production systems, this research supports the broader goals of sustainable energy utilization, agricultural self-sufficiency, and environmental protection in Kano State and Nigeria at large.

2. Literature Review

Many researchers have undertaken numerous evaluations of diverse repairable systems, employing a variety of methodologies. Some notable approaches have addressed different system configurations, modeling techniques, and statistical tools to enhance performance assessment and reliability prediction.

Ye et al. (2019) proposed an integrated model for evaluating the reliability of production systems. Their model considers the interaction between product quality and machine performance within manufacturing environments.

Singh et al. (2022) conducted a comprehensive investigation into complex systems operating under a k-out-of-n: G configuration, where consecutive degraded states occur. They employed a copula repair approach to model and assess system performance.

Ibrahim et al. (2022) explored the application of the Copula technique in evaluating the performance of an industrial system, emphasizing the importance of interdependence between system components.

Wang, Liu, Li, and Wang (2022) introduced an innovative approach to estimate the coverage factor within the context of accelerated degradation testing (ADT). ADT is a critical tool used to assess the longevity and reliability of products by exposing them to stress conditions. The coverage factor helps extrapolate degradation data from these tests to normal operating conditions. Their method enhances the accuracy and precision of reliability predictions.

Ding et al. (2021) examined the estimation of coverage factors and credible intervals in degradation tests with unknown covariance structures. They addressed challenges posed by the lack of known relationships between variables, aiming to improve estimation methods in such complex scenarios.

Zhang et al. (2020) employed Bayesian methods to forecast coverage factors in accelerated degradation testing. Their framework provided robust tools for analyzing degradation data and projecting system reliability.

Chen et al. (2020) introduced a method to estimate coverage factors by leveraging reliability analysis. Their study emphasized key aspects of reliability and contributed significantly to refining techniques for product performance assessment under accelerated conditions.

Hu et al. (2019) developed a pioneering approach for estimating coverage factors in degradation tests with incomplete data, addressing a common challenge in real-world reliability studies.

Sha (2021) conducted research on hybrid systems using a copula-based reliability analysis approach, offering insights into the interactions between diverse system components.

Tyagi et al. (2021) analyzed repairable parallel systems with fault coverage using copula-based measures, providing a statistical foundation for assessing system resilience and fault tolerance.

Ram et al. (2023) carried out a detailed analysis of series-parallel systems, focusing on their sensitivity to component failures. Their work highlighted how different failure scenarios from isolated faults to cascading failures can affect overall system functionality.

Maihulla et al. (2023) studied the reliability and performance of series-parallel photovoltaic (PV) systems, incorporating human operator roles. They used the Gumbel-Hougaard family copula to model the interactions between system components, including PV units and human inputs.

Similarly, Isa et al. (2023) assessed the performance of series-parallel computer systems by modeling the influence of human operators using the same copula framework.

Ayagi et al. (2023) presented an extensive study on performance modeling of serial systems operating under a k-out-of-n: G scheme. Their analysis considered the additional complexity of controller failure, examining its effects on system reliability.

Musa et al. (2023) performed a comprehensive reliability analysis of a small-scale residential solar system. Their study evaluated performance, durability, and potential failure modes, considering factors like component lifespan and environmental impacts.

Isa et al. (2024) conducted a detailed performance estimation of honeynet systems aimed at enhancing network security. They utilized a copula-based linguistic approach to analyze how these deceptive environments can detect and study cyber threats.

Finally, Zarogi et al. (2024) investigated the dependability of a hybrid multi-client computer networking system. Their study employed the Gumbel-Hougaard family copula to model interdependencies among clients and assess system reliability.

3. Methodology

This section describe the methodology employed in the study, including the model assumptions, notations, and description.

3.1. Notations, assumptions, and Model Description

3.1.1 Notations

t: Variable time on a time scale. s: Variable for the Laplace transform over all expressions.

ϑ_1: Unit rate of failure in tanks 1 and 2 for ammonia.

ϑ_2: Unit rate of failure in utility thermal and cooling units.

ϑ_3: Rate of failure of Green Ammonia plant.

ϑ_4: Rate of failure of Green Urea plant.

$r(x)/r(y)$: Repair rate of tanks 1 and 2 for ammonia/repair rate of utility thermal and cooling units.

$r_0(x)/r_0(y)/r_0(m)/r_0(n)$: repair rate for completely failed states of ammonia tanks 1 and 2/utility thermal and cooling units /Green ammonia plant/Green urea plant.

$N_i(t)$: For $i = 0$ to 8, the probability that a system is always in S_i state at any given period of time. $N(s)$: State transition probability under the Laplace transformation N(t).

$N_i\ (x,\ t)$: The probability that a system is in state Si , such that for $i = 1$ ..., the system is undergoing repair, and the amount of time that has passed since the start of repair is given by $(x,\ t)$ where x is the repair and t is the time variable.

$N_i\ (y,\ t)$: The probability that a system is in state $S\ i$, such that for $i = 1$ ..., the system is under repair, and the elapsed repair time is $(y,\ t)$ with y being the repair variable and t being the time variable.

$N_i\ (m, t)$: The probability that a system is in state $S\ i$ for $i = 1$, the system is undergoing repair, and the elapsed repair time is given by the expression (m, t) with repair variable being m and time variable being t.
$N_i\ (n, t)$: The probability that a system is in state S i for $i = 1$, the system is under repair, and the elapsed repair time is given by (m, t) with repair variable m and time variable t.

$E_p\ (\ t)$: Profit anticipated over the time period $[0, t)$.

P_1, P_2: Revenue and service cost per unit time, respectively.

3.1.2. Assumptions

1. All subsystems and components are initially considered to be operational.
2. Two Ammonia Storage units (tanks), two utility thermal and cooling units, an ammonia plant, and a urea plant are all required for system operation.
3. Performance of the system is satisfactory when any unit fails.
4. Whether in use or in the failure condition, any failing parts or units can be repaired.
5. It is assumed that all failure rates are constant and have an exponential distribution.
6. In contrast to General distribution, which can restore partially failed states, the Gumbel–Hougaard Family Copula can restore entirely failed states.
7. The repair component or unit must perform flawlessly, and no damage should have been done during the repair process.

3.1.3. Model Description

In this solar-integrated configuration (Figure 1), green ammonia is synthesized using hydrogen produced through water electrolysis powered by solar energy, while nitrogen is supplied by the air separation unit (ASU). The ammonia generated is subsequently reacted with renewable carbon dioxide to produce green urea in the urea plant. The solar steam and thermal utility units supply process heat, while the solar power and cooling units provide the necessary electrical and thermal control for stable operation. An energy storage system is incorporated to ensure continuous operation during periods of low solar irradiance.

From a reliability perspective, the entire system operates as a series-network, where the reliability of each subsystem directly influences overall system performance. System failure is defined as the failure of either the green ammonia or green urea plant, or the simultaneous failure of all active units in both the ammonia storage and utility subsystems. To capture real operational conditions, lethal and nonlethal failure modes are considered, along with coverage factors that represent the system's ability to detect, isolate, and recover from partial failures.

The reliability evaluation and optimization of this solar-based ammonia–urea production system thus focus on enhancing redundancy, improving fault coverage, and optimizing maintenance and energy allocation strategies. This comprehensive approach ensures high operational availability, improved fault tolerance, and sustainable production of ammonia and urea with minimal environmental impact.

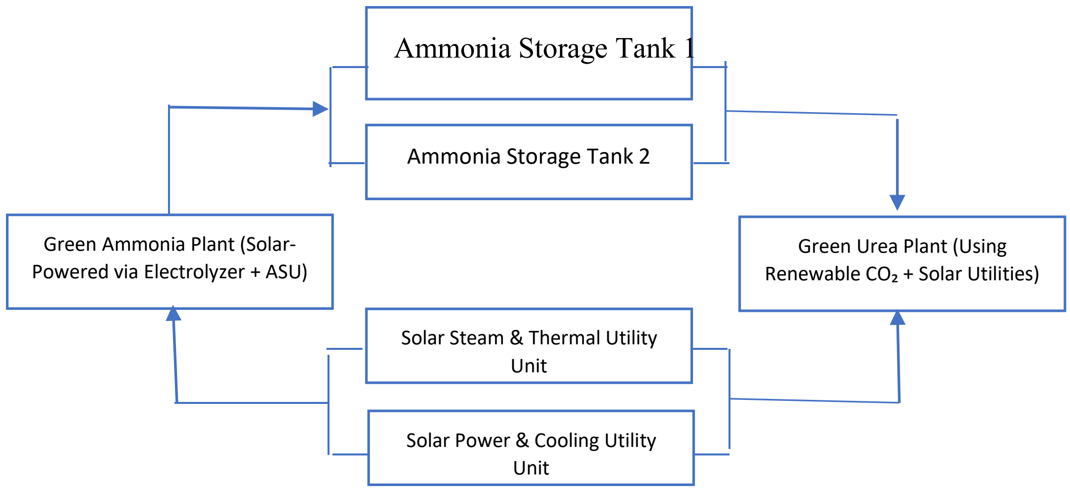

Figure 1: Block Diagram of Model

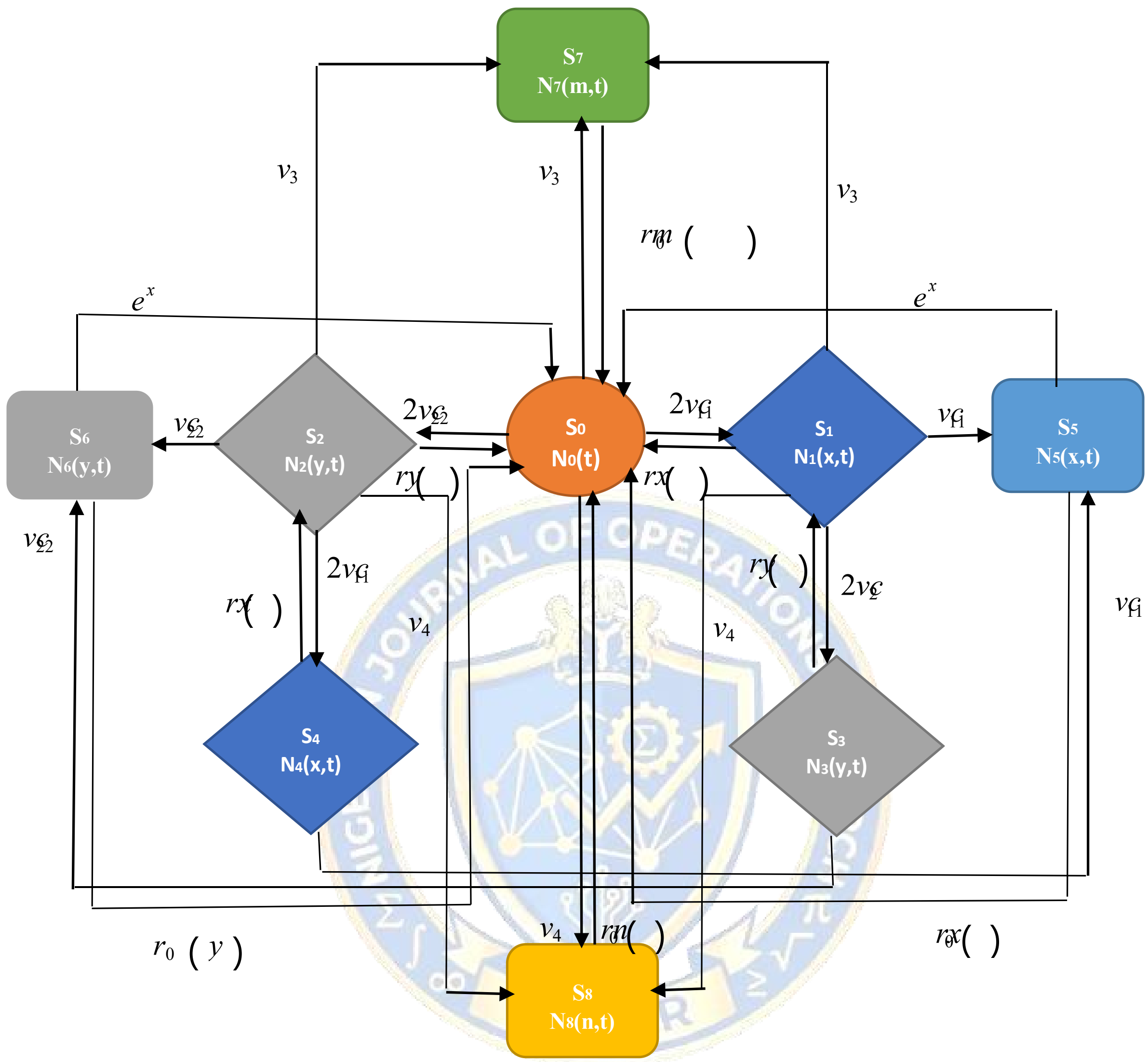

Figure 2: Transition Diagram of the Model

3.2. Model Formulation

If the Mnemonic rule is used to build model equations at every state S_i for some $i = 0,1,2,\ldots 8$ the system of first order differential equations can be determined using the state transition diagram in figure 2.

$$\left(\frac{\partial}{\partial t}+2v_1c_1+2v_2c_2+v_3+v_4\right)N_0(t)=\int_0^\infty r(x)N_1(x,t)dx+\int_0^\infty r(y)N_2(y,t)dy+\int_0^\infty \left(r_0(x)+e^x\right)N_5(x,t)dx+$$
$$\int_0^\infty\left(r_0(y)+e^x\right)N_6(y,t)dy+\int_0^\infty r_0(m)N_7(m,t)dm+\int_0^\infty r_0(n)N_8(n,t)dn \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\nu_1 c_1+2\nu_2 c_2+\nu_3+\nu_4+r(x)\right)N_1(x,t)=0, \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\nu_1 c_1+\nu_2 c_2+\nu_3+\nu_4+r(y)\right)N_2(y,t)=0, \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\nu_2 c_2+r(y)\right)N_3(y,t)=0, \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\nu_1 c_1+r(x)\right)N_4(x,t)=0, \quad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(r_0(x)+e^x\right)\right)N_5(x,t)=0, \quad (6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\left(r_0(y)+e^x\right)\right)N_6(y,t)=0, \quad (7)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+r_0(m)\right)N_7(m,t)=0, \quad (9)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+r_0(n)\right)N_8(n,t)=0. \quad (10)$$

1.1.1. Boundary conditions:

$$N_1(0,t)=2\nu_1 c_1 N_0(t), \quad (11)$$

$$N_2(0,t)=2\nu_2 c_2 N_0(t), \quad (12)$$

$$N_3(0,t)=4\nu_1\nu_2 c_1 c_2 N_0(t), \quad (13)$$

$$N_4(0,t)=4\nu_1\nu_2 c_1 c_2 N_0(t), \quad (14)$$

$$N_5(0,t)=2(\nu_1 c_1)^2\{1+2\nu_2 c_2\}N_0(t), \quad (15)$$

$$N_6(0,t)=2(\nu_2 c_2)^2\{1+2\nu_1 c_1\}N_0(t), \quad (16)$$

$$N_7(0,t) = v_3\{1 + 2v_1c_1 + 2v_2c_2\}N_0(t), \tag{17}$$

$$N_8(0,t) = v_4\{1 + 2v_1c_1 + 2v_2c_2\}N_0(t). \tag{18}$$

3.2.2. Model solution

Taking the Laplace transformation of Equations (1) to (10) with the help of initial condition $N_0(0) = 1$, one may obtain:

$$(s + 2v_1c_1 + 2v_2c_2 + v_3 + v_4)\overline{N_0}(s) = \int_0^\infty r(x)\overline{N_1}(x,s)\,dx + \int_0^\infty r(y)\overline{N_2}(y,s)\,dy + \int_0^\infty (r_0(x) + e^x)\overline{N_5}(x,s)\,dx + \int_0^\infty (r_0(y) + e^x)\overline{N_6}(y,s)\,dy + \int_0^\infty r_0(m)\overline{N_7}(m,s)\,dm + \int_0^\infty r_0(n)\overline{N_8}(n,s)\,dn, \tag{19}$$

$$\left(s + \frac{\partial}{\partial x} + v_1c_1 + 2v_2c_2 + v_3 + v_4 + r(x)\right)\overline{N_1}(x,s) = 0, \tag{20}$$

$$\left(s + \frac{\partial}{\partial y} + 2v_1c_1 + v_2c_2 + v_3 + v_4 + r(y)\right)\overline{N_2}(y,s) = 0, \tag{21}$$

$$\left(s + \frac{\partial}{\partial y} + v_2c_2 + r(y)\right)\overline{N_3}(y,s) = 0, \tag{22}$$

$$\left(s + \frac{\partial}{\partial x} + v_1c_1 + r(x)\right)\overline{N_4}(x,s) = 0, \tag{23}$$

$$\left(s + \frac{\partial}{\partial x} + (r_0(x) + e^x)\right)\overline{N_5}(x,s) = 0, \tag{24}$$

$$\left(s + \frac{\partial}{\partial y} + (r_0(y) + e^x)\right)\overline{N_6}(y,s) = 0, \tag{25}$$

$$\left(s + \frac{\partial}{\partial m} + r_0(m)\right)\overline{N_7}(m,s) = 0, \tag{26}$$

$$\left(s + \frac{\partial}{\partial n} + r_0(n)\right)\overline{N_8}(n,s) = 0. \tag{27}$$

Taking Laplace transforms of the boundary conditions presented in equations (11) to (18), we obtain:

$$\overline{N_1}(0,s) = 2\nu_1 c_1 \overline{N_0}(s), \tag{28}$$

$$\overline{N_2}(0,s) = 2\nu_2 c_2 \overline{N_0}(s), \tag{29}$$

$$\overline{N_3}(0,s) = 4\nu_1 \nu_2 c_1 c_2 \overline{N_0}(s), \tag{30}$$

$$\overline{N_4}(0,s) = 4\nu_1 \nu_2 c_1 c_2 \overline{N_0}(s), \tag{31}$$

$$\overline{N_5}(0,s) = 2(\nu_1 c_1)^2 \{1 + 2\nu_2 c_2\} \overline{N_0}(s), \tag{32}$$

$$\overline{N_6}(0,s) = 2(\nu_2 c_2)^2 \{1 + 2\nu_1 c_1\} \overline{N_0}(s), \tag{33}$$

$$\overline{N_7}(0,s) = \nu_3 \{1 + 2\nu_1 c_1 + 2\nu_2 c_2\} \overline{N_0}(s), \tag{34}$$

$$\overline{N_8}(0,s) = \nu_4 \{1 + 2\nu_1 c_1 + 2\nu_2 c_2\} \overline{N_0}(s). \tag{35}$$

Now solving Equations (**1**9) to (27), with the help of Equations (28) to (35), yields

$$\overline{N_0}(s) = \frac{1}{A}, \tag{36}$$

$$\overline{N_1}(s) = \frac{2\nu_1 c_1}{A}\left\{\frac{1-\overline{S_r}(s+\nu_1 c_1 + 2\nu_2 c_2 + \nu_3 + \nu_4)}{s+\nu_1 c_1 + 2\nu_2 c_2 + \nu_3 + \nu_4}\right\}\overline{N_0}(s), \tag{37}$$

$$\overline{N_2}(s) = \frac{2\nu_2 c_2}{A}\left\{\frac{1-\overline{S_r}(s+2\nu_1 c_1 + \nu_2 c_2 + \nu_3 + \nu_4)}{s+2\nu_1 c_1 + \nu_2 c_2 + \nu_3 + \nu_4}\right\}\overline{N_0}(s), \tag{38}$$

$$\overline{N_3}(s) = \frac{4\nu_1 \nu_2 c_1 c_2}{A}\left\{\frac{1-\overline{S_r}(s+\nu_2 c_2)}{s+\nu_2 c_2}\right\}\overline{N_0}(s), \tag{39}$$

$$\overline{N_4}(s) = \frac{4\nu_1 \nu_2 c_1 c_2}{A}\left\{\frac{1-\overline{S_r}(s+\nu_1 c_1)}{s+\nu_1 c_1}\right\}\overline{N_0}(s), \tag{40}$$

$$\overline{N_5}(s) = \frac{2(\nu_1 c_1)^2(1+2\nu_2 c_2)}{A}\left\{\frac{1-\overline{S_{(r_0+e^x)}}(s)}{s}\right\}\overline{N_0}(s), \tag{41}$$

$$\overline{N_6}(s) = \frac{2(\nu_2 c_2)^2(1+2\nu_1 c_1)}{A}\left\{\frac{1-\overline{S_{(r_0+e^x)}}(s)}{s}\right\}\overline{N_0}(s), \tag{42}$$

$$\overline{N_7}(s) = \frac{\nu_3(1+2\nu_1 c_1+2\nu_2 c_2)}{A}\left\{\frac{1-\overline{S_{r_0}}(s)}{s}\right\}\overline{N_0}(s), \tag{43}$$

$$\overline{N_8}(s) = \frac{\nu_4(1+2\nu_1 c_1+2\nu_2 c_2)}{A}\left\{\frac{1-\overline{S_{r_0}}(s)}{s}\right\}\overline{N_0}(s). \tag{44}$$

Where,

$$A = (s+2\nu_1 c_1+2\nu_2 c_2+\nu_3+\nu_4) - \left\{\begin{array}{l} 2\nu_1 c_1 \overline{S_r}(s+\nu_1 c_1+2\nu_2 c_2+\nu_3+\nu_4)+2\nu_2 c_2 \overline{S_r}(s+2\nu_1 c_1+\nu_2 c_2+\nu_3+\nu_4)+ \\ 2(\nu_1 c_1)^2(1+2\nu_2 c_2)\overline{S_{(r_0+e^x)}}(s)+2(\nu_2 c_2)^2(1+2\nu_1 c_1)\overline{S_{(r_0+e^x)}}(s)+ \\ \nu_3(1+2\nu_1 c_1+2\nu_2 c_2)\overline{S_{r_0}}(s)+\nu_4(1+2\nu_1 c_1+2\nu_2 c_2)\overline{S_{r_0}}(s) \end{array}\right\}.$$

The Laplace transformations of the state transition probabilities that the system is in operational mode, i.e. perfect and partially failed state at any time are as follows:

$$\overline{N_{up}}(s) = \overline{N_0}(s)+\overline{N_1}(s)+\overline{N_2}(s)+\overline{N_3}(s)+\overline{N_4}(s). \tag{45}$$

$$\overline{N_{down}}(s) = 1-\overline{N_{up}}(s). \tag{46}$$

3.3. Analytical solution for particular cases

3.3.1. System Availability Analysis

To obtain the availability of the system, we set $\bar{S}_r(s) = \frac{r}{s+r}, \frac{1-\bar{S}_r(s)}{s} = \frac{1}{s+r}$,

$\bar{S}_{r_0}(s) = \bar{S}_{exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}(S) = \frac{exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}{s+exp\left[x^\theta+\{log\emptyset(x)\}^\theta\right]^{1/\theta}}$, $\nu_1 = 0.01$, $\nu_2 = 0.02$, $\nu_3 = 0.03$, $\nu_4 = 0.04$ and all repairs to 1. i.e. $r(x) = r(y) = r_0(x) = r_0(y) = r_0(m) = r_0(n) = 1$ in (45).

Then taking the inverse Laplace transform, the expressions for system availability for both Copula and General repair policies can be obtained as follows:

3.3.2. Copula Repair Policy

$$N_{up}(t) = 1.89833465 10^{-10} e^{-5.436600001t} - 8.17771000910 10^{-10} e^{-1.000010000t} + 0.01123799079 e^{-2.7487458945892t}$$
$$-0.000002677139890 e^{-1.070100912t} - 2.01959590110^{-10} e^{-1.070046257t} + 0.9887647082 e^{-0.03955693816t} -$$
$$8.1776230810^{-10} e^{-1.000020000t} \tag{47}$$

For various values of variable time $t = 0, 1, 2, 3{,}4, 5, 6, 7, 8, 9{,}10$, units of time and with the aid of equation (47), different values of $N_{up}(t)$ without Coverage factor and with Coverage factor, respectively for Copula repair can be obtained as shown in Tables 1 and 2.

Table 1: Availability variance with respect to time for Copula Repair without Coverage factor

Time	0	1	2	3	4	5	6	7	8	9	10
N_{up}	1.0000	0.9526	0.9151	0.8789	0.8443	0.8115	0.7890	0.7496	0.7208	0.6932	0.6659
N_{down}	0.0000	0.0492	0.0882	0.1229	0.1579	0.1889	0.2801	0.2604	0.2895	0.3174	0.3443

Table 2: Availability variance with respect to time for Copula Repair with Coverage factor

Time	0	1	2	3	4	5	6	7	8	9	10
N_{up}	1.0000	0.9511	0.9136	0.8781	0.8441	0.8113	0.7799	0.7491	0.7205	0.6926	0.6657
N_{down}	0.0000	0.0489	0.0864	0.1219	0.1559	0.1887	0.2201	0.2504	0.2795	0.3074	0.3343

3.4. General Repair

$$N_{up}(t) = 2.66733412910_{-13}e_{-1.000010000t} + 0.00007803699183e_{-1.070148374t} + 1.68132225210_{-\,-7}\,e_{1.070046448t}$$

$$+0.03136917598e_{-1.031165872t} + 0.9685526190e_{-0.03878930571t} + 5.33632440310_{-13}e_{-1.000020000t}. \quad (49)$$

For different values of variable time $t = 0, 1, 2, 3, 4{,}5, 6, 7, 8, 9, 10$, units of time and with the help of equation (48), different values of $N_{up}(t)$ without Coverage factor and with Coverage factor for General repair policy can be obtained as shown in Tables 3 and 4, respectively below.

Table 3: Availability variance with respect to time for General Repair without Coverage factor

Time	0	1	2	3	4	5	6	7	8	9	10
N_{up}	1.0000	0.9429	0.9003	0.8636	0.8299	0.7980	0.7675	0.7383	0.7102	0.6831	0.6571
N_{down}	0.0000	0.0571	0.0997	0.1364	0.1701	0.2020	0.2325	0.2617	0.2898	0.3169	0.3429

Table 4: Availability variance with respect to time for General Repair with Coverage factor

Time	0	1	2	3	4	5	6	7	8	9	10
N_{up}	1.0000	0.9425	0.9000	0.8633	0.8296	0.7979	0.7660	0.7376	0.7003	0.6825	0.6564
N_{down}	0.0000	0.0569	0.0994	0.1360	0.1601	0.2023	0.2225	0.2517	0.2895	0.3159	0.3324

3.5. Reliability Analysis

In reliability analysis, we set all repair rates in equation (47) to zero with same values of failure rates $\nu_1 = 0.01$, $\nu_2 = 0.02$, $\nu_3 = 0.03$, $\nu_4 = 0.04$ and then take the inverse Laplace transform to obtain the expression for system reliability as:

$$R(t) = 1.000000004e^{-0.07000000260t} - 2.60641667010^{-9} e^{1.070103720t} + 5.539434813110^{-12} e^{-1.070046277t}$$

$$-8.59460154610^{-10} e^{-1.000020000t} - 8.59469501010^{-10} e^{-1.000010000t}. \quad (50)$$

Different reliability values can be obtained for different values of variable time $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$, units of time with the help of equation (50) as shown in Tables 5 and 6.

Table 5: Reliability variance with respect to time without Coverage factor

Time	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Reliability	1.0000	0.9328	0.8694	0.8106	0.7558	0.7047	0.657	0.6126	0.5712	0.5326	0.4966

Table 6: Reliability variance with respect to time with Coverage factor

Time	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Reliability	1.0000	0.9324	0.8691	0.8095	0.7538	0.7019	0.6536	0.6086	0.5667	0.5277	0.4914

3.6. Sensitivity Analysis

The MTTF sensitivity of the system is obtained by partially differentiating the MTFF with respect to the system failure rates. The MTFF sensitivity can be obtained as shown in table 7 by applying the set of parameters as $\nu_1 = 0.01$, $\nu_2 = 0.02$, $\nu_3 = 0.03$, $\nu_4 = 0.04$ in the partial differentiation of MTTF.

Table 7: Computation of MTTF sensitivity against the values of failure rate

Failure Rate	**Sensitivity w.r.t ϑ_1**	**Sensitivity w.r.t ϑ_2**	**Sensitivity w.r.t ϑ_3**	**Sensitivity w.r.t ϑ_4**	**Sensitivity w.r.t $C_1 = C_2$**
0.001	-0.0565	-0.0567	-400.0466	-0.5646	-625.072
0.002	-0.0563	-0.0566	-277.8104	-0.5579	-400.0466
0.003	-0.0562	-0.0564	-204.1058	-0.5511	-277.8104
0.004	-0.0560	-0.0563	-156.2685	-0.5444	-204.1058
0.005	-0.0559	-0.0561	-123.4715	-0.5377	-156.2685
0.006	-0.0557	-0.0560	-100.0119	-0.5311	-123.4715
0.007	-0.0556	-0.0558	-82.6545	-0.5244	-100.0119
0.008	-0.0554	-0.0557	-69.4527	-0.5178	-82.6545
0.009	-0.0553	-0.0555	-59.1787	-0.5113	-69.4527
0.010	-0.0551	-0.0554	-51.0265	-0.5047	-59.1787

3.6.1. Cost Analysis

If the service facility is always open, then the formula below will obtain the expected profit during the interval [0, t).

$$E_p(t) = P_1 \int_0^t N_{up}(t)dt - P_2 t, \tag{51}$$

Where P_1 and P_2 in the interval [0, t) are the revenue generated and service cost per unit time.

Equations (52) and (53) can be obtained for the same set of parameters used in obtaining system availability.

3.6.2. Cost Analysis for Copula Repair

$$cost = -0.00008e^{-3.719394113t} - 0.0045e^{-2.751688096t} + 0.0043e^{-1.168517076t} + 0.000002e^{-1.116260205t} - 24.3862e^{-0.04074051016t} + 0.0005e^{-1.010000000t} + 0.0005e^{-1.000020000t} + 25.3854 - P_2 t \tag{52}$$

Setting $P_1 = 1$ and $P_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 and varying t units of time as 0, 1, 2,3, 4, 5, 6, 7, 8,9, 10 respectively, the estimated benefit results for Copula repair in the absence of Coverage factor and presence of Coverage can be obtained as shown in Tables 8 and 9, respectively.

Table 8: Profit computation against Time for Copula Repair in the absence of Coverage Factor

Time	$E_p(t)$ $P_2 = 0.1$	$E_p(t)$ $P_2 = 0.2$	$E_p(t)$ $P_2 = 0.3$	$E_p(t)$ $P_2 = 0.4$	$E_p(t)$ $P_2 = 0.5$	$E_p(t)$ $P_2 = 0.6$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8742	0.7742	0.6742	0.5742	0.4742	0.3742
2	1.7080	1.5080	1.3080	1. 1080	0. 9080	0. 7080
3	2.5050	2.2050	1.9050	1.6050	1.3050	1.0050
4	3.2664	2.8664	2.4664	2.0664	1.6664	1.2664
5	3.9935	3.9935	2.9935	2.9935	1.9935	1.4935
6	4.6876	4.0876	3.4876	2.8876	2.2876	1.6876
7	5.3500	4.6500	3.9500	3.2500	2.5500	1.8500
8	5.9820	5.1820	4.3820	3.5820	2.7820	1.9820
9	6.5848	5.6848	4.7848	3.8848	2.9848	2.0848
10	7.1595	6.1595	5.1595	4.1595	3.1595	2.1595

Table 9: Profit computation against Time for Copula Repair in the presence of Coverage Factor

Time	$E_p(t)$ $P_2 = 0.1$	$E_p(t)$ $P_2 = 0.2$	$E_p(t)$ $P_2 = 0.3$	$E_p(t)$ $P_2 = 0.4$	$E_p(t)$ $P_2 = 0.5$	$E_p(t)$ $P_2 = 0.6$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8733	0.7733	0.6733	0.5733	0.4733	0.3733
2	1.7054	1.5054	1.3054	1. 1054	0. 9054	0. 7054
3	2.5011	2.2011	1.9011	1.6011	1.3011	1.0011
4	3.2621	2.8621	2.4621	2.0621	1.6621	1.2621
5	3.9897	3.4897	2.9897	2.4897	1.9897	1.4897
6	4.6852	4.0852	3.4852	2.8852	2.2852	1.6852
7	5.3498	4.6498	3.9498	3.2498	2.5498	1.8498
8	5.9848	5.1848	4.3848	3.5848	2.7848	1.9848
9	6.5913	5.6913	4.7913	3.8913	2.9913	2.0913
10	7.1704	6.1704	5.1704	4.1704	3.1704	2.1704

3.6.3. General Repair

$$cost = -0.0080e^{-1.107490197t} - 0.000007e^{-1.089990421t} - 0.0216e^{-1.023874097^{t}} - 2.8311 \times 10^{-7}e^{-1.000993246t} - 25.0060e^{-0.03875203829t} - 5.2331 \times 10^{-10}e^{-1.000020000t} - 4.2214 \times 10^{-7}e^{-1.010000000t} + 25.0356 - P_2 t \quad (53)$$

Setting $P_1 = 1$ and $P_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 and varying t units of time as 0, 1, 2,3, 4, 5, 6, 7, 8,9, 10 respectively, the estimated benefit results for General repair in the absence of Coverage factor and presence of Coverage can be obtained as shown in Tables 10 and 11, respectively.

Table 10: Profit computation against Time for General Repair in the absence of Coverage Factor

Time	$E_p(t)$ $P_2 = 0.1$	$E_p(t)$ $P_2 = 0.2$	$E_p(t)$ $P_2 = 0.3$	$E_p(t)$ $P_2 = 0.4$	$E_p(t)$ $P_2 = 0.5$	$E_p(t)$ $P_2 = 0.6$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8697	0.7697	0.6697	0.5697	0.4697	0.3697
2	1.6997	1.4908	1.2908	1. 0908	0. 8908	0. 6908
3	2.4728	2. 1728	1.8728	1.5728	1.2728	0.9728
4	3.2198	2.8198	2.4198	2.0198	1.6198	1.2198
5	3.9341	3.4341	2.9341	2.4341	1.9341	1.4341

6	4.6173	4.0173	3.4173	2.8173	2.2173	1.6173
7	5.2706	4.5706	3.8706	3. 1706	2.4706	1.7706
8	5.8953	5.0953	4.2953	3.4953	2.6953	1.8953
9	6.4925	5.5925	4.6925	3.7925	2.8925	1.9925
10	7.0631	6.0631	5.0631	4.0631	3.0631	2.0631

Table 11: Profit computation against Time for General Repair in the presence of Coverage Factor

Time	$E_p(t)$ $P_2 = 0.1$	$E_p(t)$ $P_2 = 0.2$	$E_p(t)$ $P_2 = 0.3$	$E_p(t)$ $P_2 = 0.4$	$E_p(t)$ $P_2 = 0.5$	$E_p(t)$ $P_2 = 0.6$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8702	0.7702	0.6702	0.5702	0.4702	0.3702
2	1.6919	1.4919	1.2919	1. 0919	0. 8919	0. 6919
3	2.4740	2. 1740	1.8740	1.5740	1.2740	0.9740
4	3.2202	2.8202	2.4202	2.0202	1.6202	1.2202
5	3.9330	3.4330	2.9330	2.4330	1.9330	1.4330
6	4.6138	4.0138	3.4138	2.8138	2.2138	1.6138
7	5.2639	4.5639	3.8639	3. 1639	2.4639	1.7639
8	5.8848	5.0848	4.2848	3.4848	2.6848	1.8848
9	6.4774	5.5774	4.6774	3.7774	2.8774	1.9774
10	7.0428	6.0428	5.0428	4.0428	3.0428	2.0428

4. Results Discussion

This section provides a detailed discussion of the results presented above, interpreting the observed trends and explaining their practical implications. Through this analysis, we explore how different model parameters influence system performance and identify the underlying factors contributing to changes in availability, reliability, sensitivity, and profitability.

Table 1 illustrates the variation of system availability over time under the Copula repair policy when the coverage factor is not considered. The results demonstrate a clear upward trend in availability as operational time increases. This suggests that the repair mechanism effectively restores failed components, thereby enhancing the system's operational continuity. The observed pattern indicates that the Copula repair policy contributes to stabilizing system performance by improving recovery efficiency during prolonged operation.

A comparable trend is observed in Table 2 when the coverage factor is considered. Nevertheless, the system availability is noticeably higher when the system operates outside the coverage area than when it functions under coverage conditions. This observation indicates that the presence of coverage may introduce additional operational constraints such as restricted resource allocation, signal attenuation, or increased interference that collectively reduce the system's overall availability.

Table 3 depicts the variation of system availability over time under the General Repair policy when the coverage factor is not considered. The results reveal a gradual increase in availability as operational time progresses, indicating that the repair process successfully restores the functionality of failed components. This improvement suggests that the General Repair approach enhances the system's ability to maintain operational continuity over extended periods, contributing to overall performance stability.

A similar pattern is observed in Table 4 when the coverage factor is incorporated. However, the system availability remains higher when the system operates without coverage compared to when it functions under coverage conditions. This implies that introducing coverage constraints may lead to reduced accessibility of repair resources or increased dependency on coverage-related parameters, thereby limiting system efficiency. Consequently, while the General Repair policy effectively improves recovery over time, its performance is somewhat diminished when the system is placed under coverage due to these additional operational challenges.

It is also noteworthy that, although both repair strategies demonstrate an overall improvement in availability over time, the availability achieved under the Copula repair policy is more appreciable than that under the General Repair policy. This difference suggests that the Copula repair mechanism provides a more effective restoration process, likely due to its ability to model interdependencies among components more accurately, thereby enhancing the system's resilience and operational reliability.

Table 5 and 6 present the system reliability results, respectively, without and with the coverage factor taken into account. In both scenarios, a clear downward trend in reliability is observed as operational time increases, indicating that the likelihood of system failure grows with prolonged operation. This decline is consistent with the expected behavior of aging or stressed components over time.

However, a comparison between the two cases reveals that the system maintains higher reliability when operating without coverage than under coverage conditions. This suggests that the introduction of coverage imposes additional operational constraints such as increased dependency on coverage-related resources, potential interference, or restricted repair accessibility that can accelerate the decrease in reliability.

Table 7 illustrates the sensitivity analysis results with respect to each failure parameter. The results indicate a general decrease in system sensitivity as the values of individual failure parameters change, suggesting that the system becomes less responsive to variations in these parameters over time. This trend may reflect the system's inherent robustness or the effectiveness of the implemented repair policies in mitigating the impact of component failures.

However, it is noteworthy that the system exhibits higher sensitivity with respect to the coverage factor compared to the other failure parameters. This implies that variations in coverage conditions such as restricted accessibility, signal quality, or resource allocation have a more pronounced impact on system performance than changes in individual failure rates. The finding highlights the critical role of coverage in influencing system behavior and underscores the importance of carefully considering coverage-related constraints in system design and operational planning.

Table 8 illustrates the variation of system cost over time under the Copula repair policy when the coverage factor is not considered. The results show a clear upward trend in cost as operational time increases, reflecting the cumulative expenses associated with repairs and maintenance over prolonged operation. This pattern suggests that, while the Copula repair mechanism efficiently restores failed components and maintains system continuity, it incurs increasing operational costs as the system ages or experiences more frequent failures. Nevertheless, the observed trend indicates that the Copula repair policy helps manage and stabilize costs by optimizing the repair process and reducing the likelihood of extensive failures that could be more expensive to rectify.

A comparable trend is observed in Table 9 when the coverage factor is incorporated. However, the system incurs higher costs under coverage conditions than when operating without coverage. This increase can be attributed to additional operational constraints, such as limited access to repair resources, coverage-dependent logistics, or interference effects, which may make maintenance more complex and expensive.

Table 10 depicts the variation of system cost over time under the General Repair policy when the coverage factor is not considered. The results show a steady increase in cost as operational time progresses, reflecting the expenses associated with restoring failed components. This trend indicates that the General Repair policy also effectively maintains system functionality, though with a gradual accumulation of costs over time.

A similar pattern is observed in Table 11 when the coverage factor is included. However, system costs are higher when operating under coverage compared to operating without coverage, suggesting that coverage introduces additional constraints or dependencies that increase repair and maintenance expenses. Consequently, while the General Repair policy succeeds in restoring system components, its cost efficiency is reduced under coverage conditions.

It is also important to note that, although both repair strategies experience rising costs over time, the Copula repair policy generally results in lower overall costs compared to the General Repair policy. This indicates that the Copula mechanism not only enhances system availability but also manages repair and maintenance expenditures more effectively, likely due to its ability to account for interdependencies among components and prioritize repairs more efficiently.

5. Conclusion

This study provides an in-depth analysis of the performance of a system under two repair strategies; Copula and General Repair considering the effects of coverage, operational time, and failure parameters on availability, reliability, sensitivity, and cost. The results demonstrate that both repair policies improve system performance over time, but the Copula repair policy consistently outperforms the General Repair policy across all evaluated metrics. Specifically, Copula repair achieves higher availability and reliability, exhibits lower sensitivity to individual failure parameters, and incurs lower operational costs, likely due to its ability to account for interdependencies among system components and optimize repair prioritization.

The analysis also highlights the impact of the coverage factor. While both policies improve performance, system availability and reliability are higher when operating without coverage, and costs are lower in such scenarios. Coverage introduces additional operational constraints, such as restricted access to repair resources, potential interference, or logistical challenges, which can reduce efficiency, increase expenses, and make the system more sensitive to parameter changes. The sensitivity analysis particularly emphasizes that the system is most responsive to variations in the coverage factor, underscoring its critical role in operational planning.

From a Kano state perspective, the results suggest that system performance under the Copula repair policy aligns with the "delighter" and "performance" categories enhancing customer satisfaction by both improving expected functionality (availability, reliability) and exceeding standard expectations (reduced cost and sensitivity). In contrast, the General Repair policy tends to fulfill the "must-be" requirements, maintaining baseline operational performance but not maximizing user or operational satisfaction.

5.1. Limitations

Several limitations of this study should be noted:

1. The analysis assumes deterministic failure rates and repair times; in practice, these parameters may be stochastic or influenced by external factors not considered in this model.
2. The coverage factor is modeled in a simplified manner, without accounting for dynamic network conditions or resource-sharing effects.
3. Only two repair strategies were evaluated; other advanced policies or hybrid approaches might yield different insights.
4. The cost analysis focuses on repair and maintenance expenses but does not include broader operational or downtime costs.

5.2. Future Research Directions

Future studies can build on this work by:

1. Incorporating stochastic modeling for failure, repair, and coverage conditions to reflect real-world uncertainties.
2. Exploring hybrid repair strategies that combine elements of Copula and General Repair to optimize both availability and cost.
3. Examining the dynamic effects of coverage, including mobile or adaptive coverage networks, on system performance and sensitivity.
4. Extending the analysis to multi-objective optimization, balancing availability, reliability, cost, and customer satisfaction metrics in real-time decision-making.
5. Investigating the application of machine learning techniques to predict failures and dynamically allocate repair resources, potentially enhancing the benefits observed under the Copula repair policy.

Acknowledgements

The authors wish to thank Tertiary Education Trust Fund (TETFund) for their support in this research.

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